Significant Occurrence in Even Musical Texture in Bach's Preludes, a Study Using Mathematical Tools.

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Since Pythagoras' days and even earlier, 'Learned Schemata' (schemata that are culture dependent, which include: intervals, harmonies, scales and tonal organization), were studied a lot, but only lately the interest in 'Natural Schemata' [schemata that are not culture dependent, include: curves of different parameters (pitch, intensity, rhythm etc.), operations on units (as sequences, enlargement, mirror, etc.) and more], is growing (see: Adams 1976, [1]; Huron 2000, [3]; D.Cohen 1997, [2] and others).

To our best knowledge, in all previous researches, the two schemata seemed unconnected till now.

Our study focuses on hidden significant occurrences of natural schemata in '**even**' musical texture (see definitions bellow), mostly via preludes from 'Well-Tempered Clavier' (WTC) book I, by J.S.Bach.

The results of our research shed light on Bach's special musical language and revealed latent but robust relations between learned and natural schemata.

Two main 'even' properties appeared in all our chosen preludes:

1. Similar duration of all notes (1/16th).

2. A small defined melodic pattern that was the seed of the piece out of which Bach

created the whole structure.

Those small patterns and their deviations (related to the first pattern in each prelude), were in the basis of our study.

For better understanding of our results, here are few essential definitions:

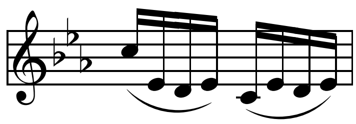
'**Evenness**': A musical property that stays constant. e.g. duration of notes, a small repeated melodic or rhythmic curve, intensity, timbre, etc.

'**patterns**': A basic element of music that has a melodic meaning, usually half a measure long between 4 to 6 notes. Each even prelude has one basic pattern which is repeated with small deviations all through the piece.

ex.

'**nuclei**': Each pattern is divided into 2 nuclei, usually symmetrically but not necessarily. (In do M the first nucleus includes 5 notes while the second one only 3).

Example of the first pattern from Do minor prelude, the referee pattern, which includes two nuclei (marked out by slurs).



In order to expose Bach's secrets, we used few mathematical tools - mainly statistical and geometric tools: Linear and Parabolic Regression, Correlation Coefficients and others (see 'Statistical Methods - The Geometry Approach' 1993, by D.Saville and G.Wood, [4]). Mathematics, represents the height of abstraction regarding rules of organization, therefore it is well-suited to expressing music.

Checking the common versus the difference, the location of small deviations of the melodic patterns, we found meaningful relations between the natural schemata and the harmony structure of the even preludes.

The research shows that "evenness" in keyboard preludes is manifested in part in prolonged repetition of short, basic **patterns**, usually half a measure long (each prelude has one pattern, divided into two "**nuclei**"), which join together to form larger units on various levels (pairs of patterns and other 2^n groups). The patterns are characterized not only by the chord (which belongs to the learned schemata) but also by somewhat latent variables, some of which are fairly regular (enabling us to speak of "deviations") and some of which are complicated: curves of change in pitch, peaks of pitch, diapason, the "center of gravity" (in other words: "internal organ point"), and so on.

In addition, there are the curves of change of these characteristics on various levels of musical organization.

The findings show interesting relationship between significant places in phenomena pertaining to these characteristics which are represented mathematically, and significant places in tonal organization, shedding additional light on the rules of organization in Bach's preludes.

The following graphs exhibit the small deviations in the melodic curves versus the main harmonies and the structure of the preludes in Do minor and in Re major. **All deviations are related to the first pattern of each prelude, the referee pattern.** If we get same deviations it means sequences (we have marked it in purple and light-blue points). The black perpendicular lines plus the numbers on top, show the harmonic structure of the piece. Almost all prominent deviations occur on meaningful harmonies. Therefore the connection between the natural and learned schemata is proved.

Remark: The graphs helped us also to find out long but not close sequences, that we couldn't see before.

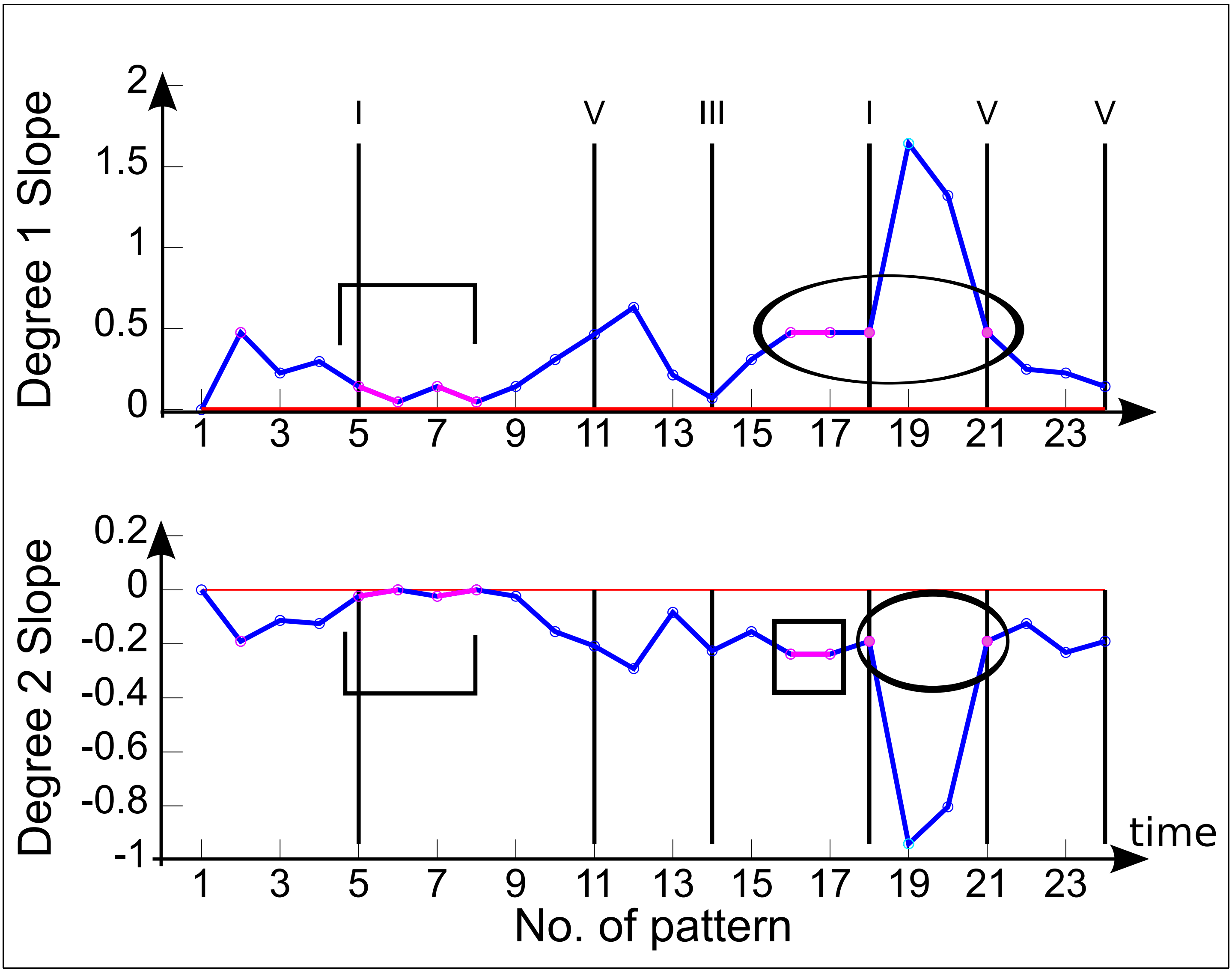


Figure 1: Parabolic versus Linear Regression in Do m

Those graphs represent only a small part of our results.

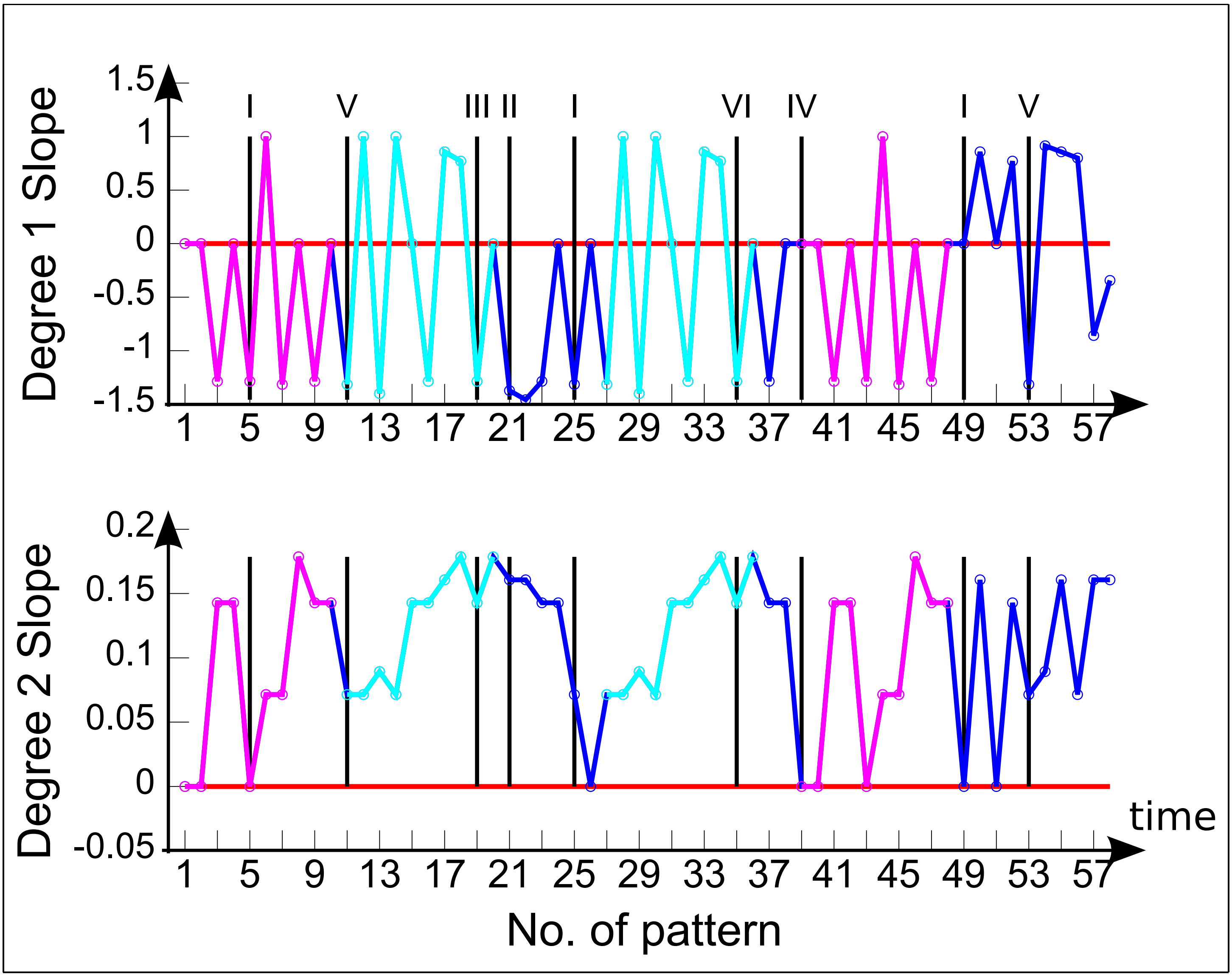


Figure 2: Parabolic versus Linear Regression in Re M

The research topic is therefore interdisciplinary, combining music and mathematics, learned and natural schemata, and explores Bach's musical language.

For comparison's sake, we can also analyze 'even' pieces by other composers and other styles (we did so to Chopin's etude no.25-1).

The findings raise some general questions, including the following:

- How is the relationship between learned and natural schemata manifested in different styles?
- How are "concurrence" and "non-concurrence" (extremely important components in shaping any style) manifested in even music?

- What are the limits to the organization of “even” flow in terms of the repeating units on the various levels?

- Artificial Intelligence: Could we ’build’ an even piece using the structure and laws we have found and defined?

- Is there any significance to “fractals” or other hidden significant structure in even music? (as suggested by one of the results of the study, based on few of the mathematical tools - “the slope of the parabolic regression ” “the slope of the correlation coefficients ” (see figures 3 , 4).

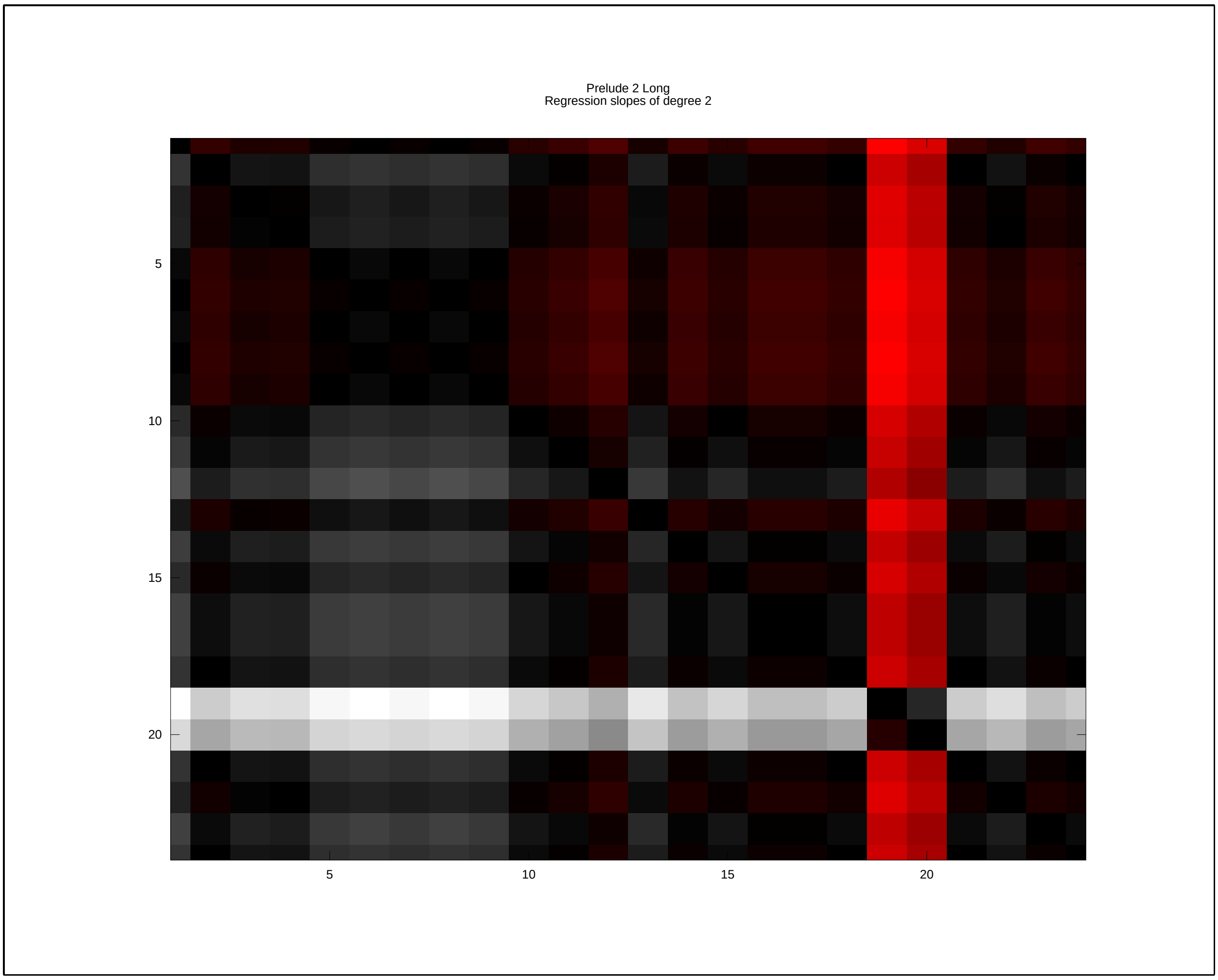


Figure 3: Graph of matrix of all parabolic regression results - Do minor

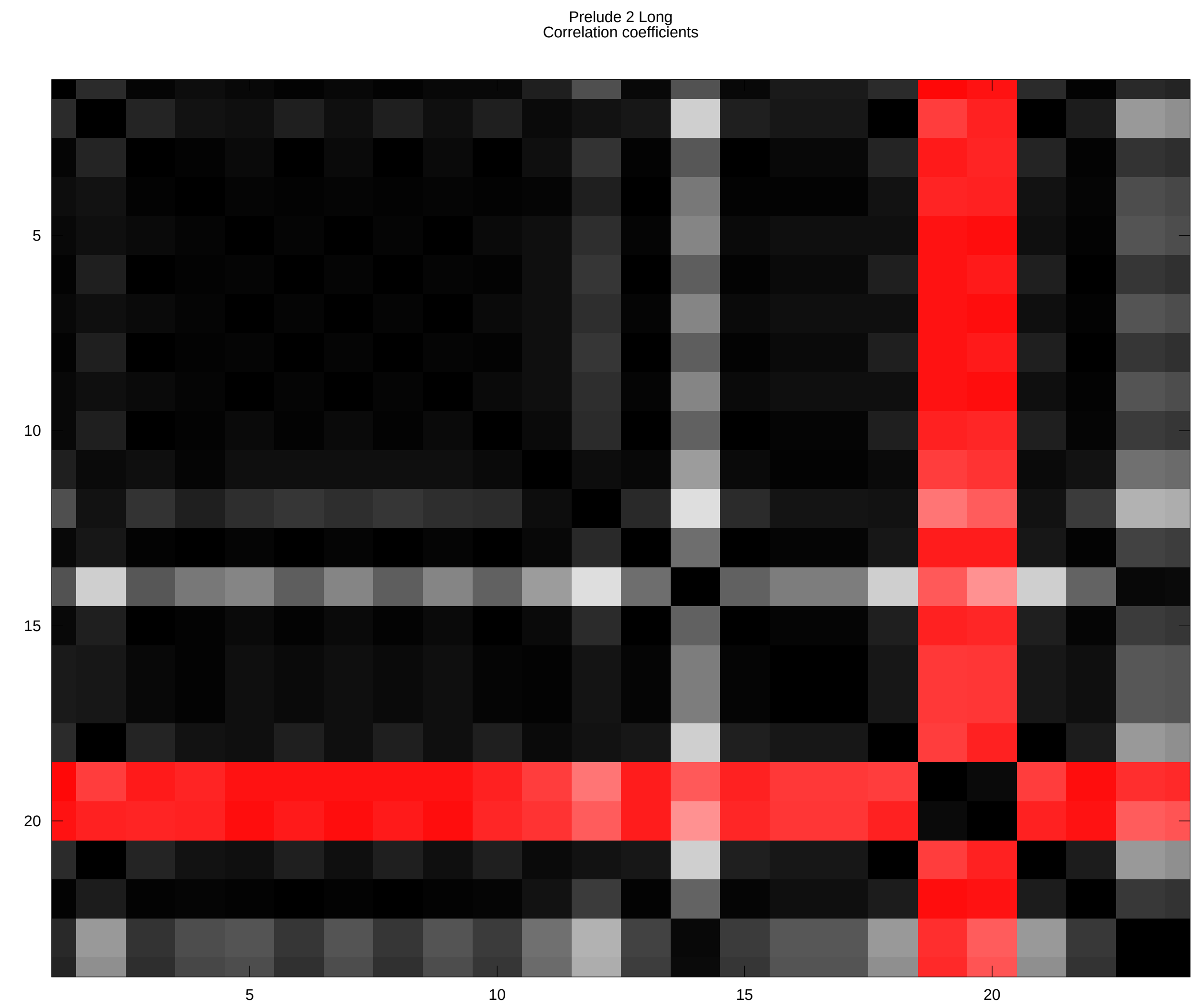


Figure 4: Graph of matrix of all correlation coefficients (Prelude 2).

The graphs above, show the small deviations in the melodic curves related to all pattern, arranged as a matrix. Black color means very similar patterns while red and white means large deviations. All large deviations appear on meaningful harmonies.

Those matrices reveal some inner structure. I would like to collaborate with a mathematician that knows some music as well, to try and find out the logic behind those matrices. Collaboration with a musician will also be welcome, in sake of analyzing music by Chopin, Schubert, etc., using the same mathematical tools,

For more information please see my PhD thesis or contact me to: 7segev@gmail.com

References

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[**2**] D. Cohen and H. Mundri. Learned amd Natural Schemata in Music. In *Proceedings of the Third Triennial ESCOM Conference, Uppsala*, pages 605–610, 1997.

[**3**] P. von Hippel and D. Huron. Why do skips precede reversals? The effect of tessitura on melodic structure. *Music Perception*, 18(1):59–85, 2000.

[**4**] D.J. Saville and G.R. Wood. *Statistical Methods: The geometric Approach.* Springer-Verlag, 1993.