

Voting on the Flag of the Weimar Republic

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Abstract

We describe and analyze the voting process leading to the compromise achieved in the Weimar Flag Controversy. We also offer a simple theoretical model that attempts to capture the main forces at work. These forces are: 1) the addition of a compromise alternative that is located between the main ideological positions on the left and on the right; 2) the interdependence of preferences that makes the compromise salient; and 3) a voting process that gradually reveals and aggregates information. Finally, we compare the theoretical insights with the observed outcome.

1 Introduction

We analyze the strategic agenda-setting and voting process that led to the compromise achieved in the Weimar Flag Controversy. We offer a simple theoretical model that

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attempts to capture the main forces that were at work in the crucial 1919 vote: 1) the construction of a "synthetic" compromise alternative that is located between the main ideological positions on the left and on the right; 2) the interdependence of preferences that makes the compromise salient; and 3) a voting process that gradually reveals and aggregates information. Finally, we compare the obtained theoretical insights to the observed outcome.

The short history of the Weimar Republic was marred by political instability and government falls, often connected to the Flag: the controversy: it represented, in compressed form, different historical and political views about the entire century preceding Weimar (see [Winkler 1993]). The principal argument was between the Black-Red-Gold (BRG) flag and the Black-White-Red (BWR) flag. The BRG flag was associated with progressive, anti-monarchistic ideas, while BWR were the official colors of the Reich in the period 1871-1919, and, significantly, already from 1867, the flag adorning of North-German Confederation's fleet.

Article 3, defining the flag, was the only article of the Weimar Constitution (out of 181!) whose outcome was determined by open voting where individual votes were registered. The Weimar constitutional assembly finally compromised on a flag that was a combination of the BWR and BRG flags. Nevertheless, the BWR flag was restored by the Nazis immediately after taking power in 1933, and later another official flag, personally designed by Hitler, combining the BWR colors and the Swastika was added.¹ BRG is again the flag of the re-united Germany, while the BWR colors are often used as a surrogate for illegal Nazi symbols. Demonstrators opposing Corona regulations attempted to storm the Reichstag in 2020 while adorning huge BWR flags. Subsequently, several German states completely banned its use.

We consider here a collective decision problem where ex ante opinions are dichotomous and cross traditional left-right party lines. This state of affairs is often observed in European parliaments – where the government is, in many cases, formed by a coal-

¹This discredited use of the BWR colors led both East and West Germany's to return to the same (!) BRG flag after WWII. At least in West Germany this decision was controversial, and many pleaded for a complete new start (see Die Zeit, 1949). East Germany added a communist emblem in 1959.

tion of several parties – whenever their members vote on major decisions that are beyond the "bread-and-butter" legislation where the government's proposal, subject to coalitional discipline, is widely anticipated to prevail. For example, in the German 2017 vote to legalize same-sex marriage, the main Government party, the CDU, was split with 225 MPs against vs. 75 in favor. The CDU and their leader Angela Merkel, who voted against, were defeated since all other parties – some of them members of the governing coalition, some in opposition – voted in favor. And in 2019, Premier May lost her job when a large group of "Brexiters" from her own party repeatedly voted against her proposal for Brexit.

In our model, several privately informed agents have single-peaked preferences such that each agent's peak is determined by his/her own signal and by the mean signal of others. Since others' signals are private information, each agent is *ex ante* uncertain about her own preferred alternative.

In addition to the two "extreme" positions on the left and on the right, we consider a compromise alternative whose location may be endogenous. In our case study, and in many other cases, the compromise is negotiated and therefore "synthetic". The interdependence of preferences is what makes the compromise salient in our model, whereas the compromise alternative would never be elected under a private values assumption. For example, during March 2019 the UK Government under Theresa May struggled to identify and approve in Parliament a compromise deal between the "hard" Brexit demanded by a vocal faction of the Tories, and the "soft" version, closer in spirit to economically remaining in the EU, supported by Labour and other smaller parties. At least 8 different compromises were suggested and tested in an "indicative" voting process.

The Weimar Assembly (as practically all modern continental European parliaments, including the EU parliament) used the *successive* voting procedure where alternatives are put to vote, one after another, until one of them gets a majority.² Agenda formation

²In contrast, all English-speaking democracies, several Scandinavian countries and Switzerland use the *amendment* voting procedure (AV) where alternatives are considered two-by-two, and where the majority winner advances to the next stage, as in an elimination tournament.

followed a long-standing practice, summarized as follows:

“If several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo.”

With interdependent values, the voting process gradually reveals and aggregates information, and agents respond to new information by adjusting their voting strategy. We relate equilibrium strategies to the degree of interdependence, to the location of the compromise, and also to the identity of the complete information Condorcet winner.

After examining the voting equilibria arising for each fixed location of the compromise, we analyze its optimal location. We first identify the main forces motivating this location, independently of the desired goal: (i) to elect an alternative that is superior to those already on the table or (ii) to insure against the election of another, worse alternative. The optimal location of the compromise is then shown to depend on several parameters such as the size and ideology of the ex ante expected majority and the degree of interdependence in preferences.

Finally, we note that our type of model and insights could illuminate other parliamentary voting procedures and other cases of voting in committee settings. For example, [Posner and Vermeule 2016] note that a more or less evenly split decision by several judges, or by a jury, may be logically incompatible with a conviction based on guilt “beyond reasonable doubt”. They propose a dynamic voting procedure where members learn about the positions of others and adjust their opinion, and also argue that a formal procedure where the revealed numbers of supporters for each option speak for themselves is better than an informal, harder to quantify deliberation.

Related Literature

Almost the entire literature on binary, sequential voting follows Farquharson [1985] and assumes that agents are completely informed about the preferences of others (see [Miller 1977], [McKelvey and Niemi 1978] and [Moulin 1979], among others, for early important contributions). In that case, the associated extensive form games can be

analyzed by backward induction: at each stage voters foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. If a simple majority is used at each stage, then, whenever it exists, a Condorcet winner is selected by sophisticated voters, independent of the particular structure of the binary voting tree, and independent of its agenda.³

An early analysis of strategic, sequential voting under incomplete information with private values is Ordeshook and Palfrey [1988]. They constructed Bayesian equilibria for an *amendment* procedure with three alternatives and with preference profiles that potentially lead to a Condorcet paradox. Gershkov, Moldovanu and Shi [2017] (GMS hereafter) analyzed voting by qualified majority in *successive voting* via a model where agents' preferences are single-peaked and follow the private values paradigm.⁴ Kleiner and Moldovanu [2017] generalized the GMS results to the class of all sequential, binary procedures with a *convex* agenda. Recall that in a binary, sequential procedure each vote is taken by (possibly qualified) majority among two, not necessarily disjoint, subsets of alternatives. Convexity says that if two alternatives a and c belong to the left (right) subset at a given node, then any alternative b such that $a < b < c$ (in the ideological order governing single-peakedness) also belongs to the left (right) subset.

Under single-peaked, private values preferences, Kleiner and Moldovanu showed that sincere voting constitutes an ex post perfect equilibrium in any voting game derived from a sequential, binary voting tree with any convex agenda.⁵ An important corollary is that, if simple majority is used at each stage of the voting tree, the associated equilibrium outcome is always the complete information Condorcet winner. Thus, all sequential binary voting trees with convex agendas and all information policies are

³If a Condorcet winner does not exist, then a member of the Condorcet cycle is elected. The influence of agenda manipulations has been emphasized by Ordeshook and Schwartz [1987], Austen-Smith [1987] and, more recently by Barbera and Gerber [2017].

⁴Their focus was on finding the welfare maximizing procedure. This is achieved by varying the thresholds needed for the adoption of each alternative.

⁵In other words, voters cannot gain by manipulating their vote, regardless of their beliefs about others' preferences, and regardless of the information disclosure policy along the voting sequence. Under a mild refinement, this equilibrium is unique.

equivalent under single-peaked, private values preferences.

Enelow [1983] contains an early model of optimal compromise location under an amendment procedure. His model is neither game-theoretic nor otherwise micro-founded: the (numerical) results depend on the agenda setter’s exogenously given beliefs about the probabilities of various outcomes.

There are only a few papers that study voting models with more than two alternatives and with interdependent values (note that interdependence generalizes the more ubiquitous assumption of common values).⁶ Gruener and Kiel [2004] and Rosar [2015] analyze static voting mechanisms in a setting where agents have interdependent preferences, focusing on the mean and the median mechanisms. Gershkov et al. [Gershkov and Kleiner and Moldovanu and Shi 2022] focus on the theoretical implementation of the Condorcet winner in such settings. Besides static mechanisms, the dynamic games they consider are variations of the amendment procedure used in Anglo-Saxon parliaments - this markedly differs from the successive procedure analyzed in the present paper. Moldovanu and Shi [2013] analyze voting in a dynamic setting where multi-dimensional alternatives appear over time and where voters are only partially informed about some aspects of the alternative. Piketty [2000] studies a two-period voting model where a large number of agents care about the decisions taken at both stages. As in our model, voting at the first stage reveals information about preferences that is relevant at the second stage. Piketty concludes that electoral systems should be designed to facilitate efficient communication, e.g. by opting for two-round rather than one-round systems—this is congruent with the kind of multi-stage procedures observed in legislatures and discussed in this paper.

Martin and Vanberg [2014] empirically test several models of legislative compromise in coalition governments, and conclude that these tend to be positions that average opinions in coalitions rather than representing, say, the view of the median coalition

⁶Dekel and Piccione [2000] analyzed sequential voting with interdependent values in a model with only two alternatives: sequentiality is with respect to individual voting. They showed that equilibrium conditioning on pivotality leads voters to ignore the revealed history. Ali and Kartik [2012] displayed other equilibria.

member. [Ezrow and De Vries and Steenbergen and Erica 2011] conducted an analysis of political parties in 15 Western European democracies from 1973 to 2003 and showed that the larger, mainstream parties tend to adjust their positions on the Left-Right spectrum in response to shifts in the position of the mean voter, while being less sensitive to policy shifts of their own supporters. The opposite pattern holds for smaller, niche parties. [Chappell 2004] studied the Federal Open Market Committee’s detailed voting patterns on monetary policy, and test the hypothesis that the chairman’s preferred policy is a weighted average of her own and the other members’ signals – the same functional form as the one adopted here.^{7,8}

The rest of the paper is organized as follows: In Section 2 we describe our case study, the voting process that determined the flag of the Weimar republic. In Section 3 we present a social choice model with interdependent preferences, and the calculation of the Condorcet winner. In Section 4 we analyze the voting equilibria under the continental successive voting procedure. Section 5 studies the optimal location of the compromise alternative. Section 6 compares the model’s theoretical predictions with the observed outcome of the case study. In Appendix A we briefly describe a probabilistic tool employed in one of the arguments. All proofs are collected in Appendix B.

2 The Case Study

The 421 seats in the Weimar National Constitutional Assembly were divided among various parties as follows:

Party	SPD	Z	DDP	DNVP	USPD	DVP	BBB	DHP	SHBLD	BL
Seats	163	91	75	44	22	19	4	1	1	1

Table 1: The division of seats among parties

⁷There are twelve members, and the chairman’s weight on his own signal is estimated to be between 0.15 and 0.20. Chappel et al. take their cue from an earlier study by Yohe [1966] who writes “...there is also no evidence to refute the view that the chairman adroitly detects the consensus of the committee, with which he persistently, in the interests of Systems harmony, aligns himself.”

⁸They also estimate the opposite influence of the chairman on members.

SPD, the left-leaning social democrats constituted the main party of the ruling coalition. Z(entrum) and DDP were centrist parties, also in the government coalition (in bold). DNVP and DVP were right-leaning conservative parties, both in the opposition. USPD, the independent social democrats were to the left of the SPD, and were also in opposition.⁹

2.1 The Proposed Flags

The Assembly considered 4 alternative proposals, 3 of them with support that crossed several party lines, leading to genuine uncertainty about the outcome of a vote:

1. BRG. This was the government's proposal, considered the "main" alternative. It was submitted to the Constitutional Committee on February 21, 1919 but it was subsequently adjusted at the initiative of the SPD to include a possible later determination of a flag for the fleet.
2. BWR. This was supported by the two opposition right-conservative parties DNVP and DVP, and by conservative factions of the centrist parties in the ruling coalition, Zentrum and DDP.
3. Red. This is the color of the Socialist International, supported by the more radical left, the USPD.¹⁰
4. BRG/BWR. This was the compromise arrangement: BRG as national colors, together with an adjusted BWR flag with BRG canton for the fleet.¹¹ The compromise was proposed by members of both Zentrum and DDP.

The clear left-right ideological order was

Red – BRG – BRG/BWR – BWR

⁹The other 4 very small parties in opposition, BBB, DHP, SHBLD and BL mostly represented regional interests.

¹⁰This proposal was also adjusted to include a provision about a future, possibly different flag for the fleet.

¹¹A canton is a small flag within a flag, usually at the NW corner.

The standard decision making procedure of the Assembly was successive voting. Explicitly mentioning the “voting on the farthest alternative first” logic, the agenda-setting Elders’ Council suggested:¹²

$$\mathbf{A} : 1) \textit{Red} \longrightarrow 2) \textit{BWR} \longrightarrow 3) \textit{BRG/BWR} \longrightarrow 4) \textit{BRG}$$

The first substantial vote was thus on Red.¹³ The rejection of Red, the preferred alternative of the radical left was widely anticipated, and this also explains why it was not deemed necessary to register individual votes in that case.¹⁴ For simplicity, we focus here on the remaining agenda

$$\mathbf{A}' : 2) \textit{BWR} \longrightarrow 3) \textit{BRG/BWR} \longrightarrow 4) \textit{BRG}$$

where the voting outcome was much more uncertain.

At each stage, each individual vote was carefully registered. First, BWR was defeated: 111 members voted in favor, 190 members voted against, and 6 abstained. Then, the compromise BRG/BWR was accepted: 211 voted in favor, 90 against and 1 abstained. According to the rules of the successive procedure, the remaining proposal BRG was not put to vote anymore.

3 A Model of Compromise

We propose below a very simple model whose major defining features are the presence of a compromise alternative that is located between the main ideological positions on the left and on the right, and the interdependence of preferences that makes this compromise salient. Such preferences can be seen as a reduced-form for indirect inter-

¹²This is the method proposed by Trendelenburg, 1850, and Tecklenburg, 1914, for cases where the proposals are on both sides of the "main" alternative, taken here to be the Government’s position. See [Protocols of the Weimar National Assembly 1919].

¹³A conservative member proposed a non-convex agenda that was defeated by simple majority before the substantial vote.

¹⁴It is probable that only members of USPD—who had proposed Red and who presumably had their peak on it—voted in its favor, while all other parties—who had peaks to the right—voted against.

actions and effects that cause agents to care about the opinion of others, e.g. the need to cooperate on other future decisions.

There are $2n + 1$ voters who collectively choose among three alternatives: L (left), C (compromise) and R (right). Let x_a denote the “location” of alternative a , $a \in \{L, C, R\}$, on a left-right ideological spectrum. The locations of the "ideological" alternatives L and R are exogenously given and normalized to be $x_L = -1$ and $x_R = +1$, while the location of the compromise, $x_C \in [-1, 1]$ may be chosen endogenously, e.g., in order to maximize some goal.

Before voting, each agent i , $i = 1, \dots, 2n + 1$, obtains a signal $s_i \in \{-1, 1\}$. Signals $\{s_i\}_{i=1}^{2n+1}$ are assumed to be i.i.d., and we let $p \in (0, 1)$ denote the ex ante probability of drawing signal -1 . Hence, voters with signal -1 are in an ex-ante minority if and only if $p < 1/2$.

We denote by \tilde{n}_{-1} the random variable representing the number of voters with signals -1 and by n_{-1} its realization. The realized number of voters with signal $+1$ is denoted by $n_{+1} = 2n + 1 - n_{-1}$. The expected number of voters with signal -1 is $E[\tilde{n}_{-1}] = (2n + 1)p$.

Each voter, $i = 1, \dots, 2n + 1$, has an “ideal” location y_i for the elected alternative. Voter i 's ideal point depends both on her own private signal s_i and also on the mean of all other voters' private signals s_j , $j \neq i$. Let $\gamma_{-1}, \gamma_1 \in [\frac{1}{2n+1}, 1]$ denote the weight that voters with signal -1 and $+1$ assign to their own signal, respectively. The ideal location $y_i(s_i, s_{-i})$ for voter i is

$$y_i(s_i, s_{-i}) = \gamma_i s_i + \frac{1 - \gamma_i}{2n} \sum_{j \neq i} s_j, \quad (1)$$

where $\gamma_i = \gamma_{-1}$ if $s_i = -1$ and $\gamma_i = \gamma_1$ if $s_i = +1$.

Thus, preferences are assumed here to be interdependent: the weight γ_i on own signal s_i , captures the level of interdependence. The weighted average form is the simplest and most commonly adopted form both in the behavioral literature (e.g. [DeGroot 1974], [Charness and Rabin 2002]) and the theoretical one (e.g. [Jehiel and Moldovanu 2000]).

In order to avoid a complex model with more than two types that determine preferences, we assumed above that the degree of interdependence in the preferences is

determined by the obtained signal. A special case is the one where all agents share a common weight $\gamma = \gamma_1 = \gamma_{-1}$. Then $\gamma = 1$ yields the private values case (no interdependence), while $\gamma = \frac{1}{2n+1}$ yields the pure common values case where, ex-post, all voters share the same ideal point.

If alternative $a \in \{L, C, R\}$ is elected, the utility of voter i with ideal point y_i is given by $u(x_a, y_i)$ where $u(\cdot, y_i)$ is single-peaked at, and symmetric around $x_a = y_i$. In particular, any utility function $u(x_a, y_i)$ that is monotonically decreasing in the absolute value of the difference between y_i and x_a is feasible. Equilibrium constructions are solely based on ordinal information.

3.1 The Condorcet Winner

An alternative is the complete information Condorcet winner if it is the Condorcet winner when all voters' types are public information. For any given realization of signals, the assumed preferences are here single-peaked according to the left-right natural orders L, C, R or R, C, L . Therefore, the Condorcet winner always exists. To compute it, note first that the ideal point of voter i with signal $+1$ can be written as

$$y_i(+1, s_{-i}) = \gamma_1 + \frac{1 - \gamma_1}{2n}(-n_{-1} + (2n - n_{-1})) = 1 - (1 - \gamma_1)\frac{n_{-1}}{n}.$$

In the private values case where $\gamma_1 = 1$, such a voter has a peak on alternative R . If $\gamma_1 < 1$, the peak monotonically shifts to the left as the number of voters with the opposite signal increases. Let

$$k = \frac{n}{2} \frac{1 - x_C}{1 - \gamma_1} \quad (2)$$

and observe that, if k is an integer and if $n_{-1} = k$, then voters with signal $+1$ are indifferent between alternatives C and R , because their peak is then given by

$$1 - (1 - \gamma_1)\frac{n}{2} \frac{1 - x_C}{1 - \gamma_1} \frac{1}{n} = \frac{1}{2}(1 + x_C),$$

exactly half-way between 1 and x_C . Thus, if $n_{-1} \leq k$, the voters with signal $+1$ form a majority, and the Condorcet winner is given by

$$CW = \begin{cases} R & \text{if } n_{-1} \leq [k] - 1 \\ C & \text{if } n_{-1} > [k] - 1 \end{cases}, \quad (3)$$

where $\lceil z \rceil$ denotes the smallest integer no less than a real number z . In this case, C can be the Condorcet winner only if $k \leq n$ which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$. If the majority group n_{+1} puts a relatively high weight on own signal, the Condorcet winner is always R .

Similarly, the ideal point of a voter i with signal -1 can be written as

$$y_i(-1, s_{-i}) = -\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(n_{+1} - (2n - n_{+1})) = -1 + (1 - \gamma_{-1})\frac{n_{+1}}{n}.$$

In the private values case where $\gamma_{-1} = 1$, such a voter has a peak on alternative L . If $\gamma_{-1} < 1$, then the peak monotonically shifts to the right as the number of voters with the opposite signal increases. Let

$$\kappa = \frac{n}{2} \frac{1 + x_C}{1 - \gamma_{-1}} \quad (4)$$

and observe that, if κ is an integer and if $n_{+1} = \kappa$, then voters with signal -1 are indifferent between L and C , because their peak is given by

$$-1 + (1 - \gamma_{-1})\frac{n_{+1}}{n} = -1 + (1 - \gamma_{-1})\frac{n}{2} \frac{1 + x_C}{1 - \gamma_{-1}} \frac{1}{n} = \frac{1}{2}(-1 + x_C),$$

exactly half way between -1 and x_C . If $n_{-1} \geq n + 1$, then voters with signal -1 form a majority, and the Condorcet winner is

$$CW = \begin{cases} L & \text{if } n_{+1} \leq \lceil \kappa \rceil - 1, \\ C & \text{if } n_{+1} > \lceil \kappa \rceil - 1 \end{cases}. \quad (5)$$

In this case, C can be the Condorcet winner only if $\kappa \leq n$ which is equivalent to $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$. If the majority group n_{-1} puts a high enough weight on own signal, then the Condorcet winner is always L .

To conclude, if the voters whose signal is in majority put a high enough weight on the opinion of others (given a fixed compromise location x_C), they will prefer the compromise alternative if and only if the number of voters with the opposite signal exceeds a certain threshold. As we shall see below, the cutoffs κ and k defined above play an important role also in the construction of strategic voting equilibria.

4 The Successive Voting Procedure

We study the equilibria of Successive Voting, as employed in the case study: alternatives are ordered according to a specific agenda, say $[L, \{C, R\}]$. With this agenda, voters first decide by simple majority to accept, or to reject alternative L . If L is accepted, voting ends. Otherwise, voters decide whether to accept alternative C . Alternative C is accepted if it has majority support and R is accepted otherwise.

A strategy profile is an ex post equilibrium if, given that all other agents follow their equilibrium strategies, each voter plays a best-response for all signal realizations. The general results of GMS and Kleiner and Moldovanu (2017) for private values and single-peaked preferences imply that this procedure yields the Condorcet winner in an ex post and sincere voting equilibrium if the agenda is either $[L, \{C, R\}]$ or $[R, \{C, L\}]$. We focus here on these agendas, where the alternative put to vote in the first stage is one of the two "extreme" alternatives. Recall that this was the rule governing agenda formation in the case study. Sincere voting need not be an equilibrium for the agenda that starts by voting on the compromise C , and, even under private values, such an agenda may not elect the Condorcet winner.

4.1 Vote Shifting and Equilibrium Characterization

We focus on the information policy that reveals the margin of victory at the first stage. The derived strategies remain an equilibrium even if individual voting behavior is reported, as long as we focus on type-symmetric equilibria where all voters with the same signal behave in the same way.¹⁵

A new important phenomenon, vote shifting, arises as a response to information disclosure and interdependent values: at the second stage, some voters condition their behavior on the voting outcome of the first stage since this past result conveys valuable

¹⁵One could also consider the minimal information policy where, if the second stage is reached, voters know only that the first stage alternative did not get the support of a majority. The policy of revealing the margin of defeat at the first stage is better than the minimal information policy in electing the Condorcet winner.

information about the signals of other agents (that directly affect their own preferences here).

Consider agenda $[L, \{C, R\}]$ and the following strategy profile: Voters with signal -1 vote in favor of L in the first stage and in favor of C at the second stage; Voters with signal $+1$ vote against L in the first stage, and in the second stage vote for C if L received at least $\lceil k \rceil$ votes in the first stage, and vote against C otherwise. We denote this profile by $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$, where the first component denotes the strategy of voters with signal -1 at stage 1 (L_1) and at stage 2 (C_2), and the second component analogously denotes the strategy of voters with signal $+1$ at the two stages. The symbol \neg denotes voting against the respective alternative.

Remarkably, the same cutoff k defined in (2) that appeared in the non-strategic determination of the complete information Condorcet winner plays a role in the strategic analysis below: it is chosen such that, when there are k voters with signal -1 , voters with signal $+1$ are indifferent between C and R . Intuitively, k is increasing in γ_1 and decreasing in x_C . That is, vote-shifting will be more likely when voters with signal $+1$ care more about other voters' private information (lower γ_1), or when the compromise is located closer to R .

With interdependent values, the successive voting procedure critically relies on vote shifting to dynamically discover the Condorcet winner. This discovery process need not be always successful, contrasting the private values case.

Proposition 1 *Consider agenda $[L, \{C, R\}]$.*

(i) *If $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$, then the strategy profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ constitutes an equilibrium that always selects the complete information Condorcet winner. If, in addition, $\gamma_1 \leq \frac{1}{2}(1 + x_C)$, actual vote-shifting may occur in equilibrium.*

(ii) *If $\gamma_{-1} < \frac{1}{2}(1 - x_C)$, then there is no equilibrium that always results in the selection of the full information Condorcet winner.*

The Condorcet winner fails to be selected in some cases: if $\gamma_{-1} < \frac{1}{2}(1 - x_C)$, voters could unanimously reject L even though L is the Condorcet winner. This cannot

happen in the private values setting.¹⁶

The results for agenda $[R, \{C, L\}]$ are analogous: consider the strategy profile

$$(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$$

where, if alternative R receives at least $\lceil \kappa \rceil$ votes in the first stage, voters with signal -1 shift and vote for C in the second stage. The cutoff κ in a vote-shifting equilibrium is the same cutoff (4) used to determine the complete information Condorcet winner. In order to have effective vote shifting in equilibrium, we need $\kappa \leq n$ which is equivalent to $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$.

Since the selection of the complete information Condorcet winner is not guaranteed, the two agendas $[L, \{C, R\}]$ and $[R, \{C, L\}]$ – that are equivalent under a private values assumption – are not anymore equivalent with interdependent values: using the characterization of equilibria for all possible parameters' values and a selection criterion (see next Section) it can be shown that the agenda that puts the "extreme" alternative with ex ante higher support last elects the Condorcet winner with a higher probability than the agenda that puts that alternative first, and we focus on it below. This order agrees well with observed practice in many parliaments, and also with the employed procedure in our case study: the alternative BRG, proposed by the majority government, was put to vote last.

The reason for the non-equivalence is connected to the direction of learning: putting the alternative with ex ante higher support first on the ballot risks of "hastily" giving up that alternative in some cases where voters rally around the compromise before anything new has been learned about the number of opponents. Indeed, if the number of opponents is relatively small, the Condorcet winner is the foregone extreme alternative, rather than the chosen compromise. Such an undesirable outcome is less likely if the first alternative on the ballot is the more extreme one with less ex ante support.

¹⁶It can be shown that this defect cannot occur in the amendment procedure with a convex agenda that always reveals information about votes received by both L and R at the first stage.

5 The Location of the Compromise

So far we have assumed that x_C , the location of the compromise alternative C on the ideological scale, is exogenous. Recalling the “synthetic” nature of compromises achieved in the political processes (such as the composite flag in our case study), we consider now the effects of varying its location. An optimal compromise location clearly depends on the underlying goal, on the expected numbers of voters with the various signals, and on the interdependence parameters. Whatever the underlying goal is, the main constraint on the optimal location is that, in order to be effective, the compromise must be elected with positive probability.

For example, the German Bundestag considered a reform of Paragraph 219a, the law governing the advertising of abortion procedures. The "extreme" alternatives were: 1) keeping the status quo that forbids any such advertising, and includes criminal charges against doctors that do so, and 2) scraping this paragraph altogether. Both the ruling coalition and the opposition contain parties on the left and on the right, and were split on this question. A compromise was forged that allows doctors and hospitals to advertise that they perform abortions, but does not allow them to provide further information about the methods, etc... The sequential voting agenda started with the two motions that wanted to scrap the law altogether. After these were defeated, the compromise was elected by a large majority.

On the other hand, Theresa May’s continued inability to get her Brexit compromise selected by the UK parliament - that ultimately caused her fall - points to a non-optimal choice of compromise, one that did not gather a majority.

For the compromise location exercise we need an intuitive and consistent method of selecting equilibria for each possible parameter constellation: the multiplicity of voting equilibria is a standard problem in voting games, and pivotality considerations alone are not sufficient for equilibrium selection. We base our selection criterion on the following intuitive concept:

Definition 1 *A voter’s strategy is sincere if, at each stage in the process, and conditional on all available information, the voter approves the current alternative if it*

yields the highest expected payoff among the remaining ones, and rejects it otherwise. A strategy profile is sincere if all agents use sincere strategies. A strategy profile is semi-sincere if one type of voters votes sincerely, but not both.

If the equilibrium strategy is not sincere, legislators may have difficulties explaining their behavior to constituents. This feature often constrains opportunistic equilibrium behavior and is the subject of a large literature in Political Science (see [Fenno 1978]). Semi-sincerity is needed for a special case where sincere equilibria do not exist.

In this Section we make the following assumption:

Assumption A Ex ante, voters with signal -1 are in minority (i.e., $p < 1/2$), and weakly prefer L to R .

Assumption A has two parts. The first part, $p < 1/2$ is without loss of generality. The second part assumes that, ex-ante, there is indeed a conflict of interest between the two types of voters (otherwise the situation is trivial). Formally, it requires that

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n} (2n(1 - p) - 2np) \leq 0 \Leftrightarrow \gamma_{-1} \geq \frac{1 - 2p}{2(1 - p)}.$$

Note that under Assumption A, the convex agenda that puts the alternative with ex-ante higher support last is $[L, \{C, R\}]$.

Let us define cutoffs γ_1^* and γ_{-1}^* as follows:

$$\begin{aligned} \gamma_{-1}^* &\equiv \frac{1}{2}(1 - x_C) + \frac{1 - 2p}{2(1 - p)} \frac{1}{2}(1 + x_C), \\ \gamma_1^* &\equiv \frac{1}{2}(1 + x_C) - \frac{1 - 2p}{2p} \frac{1}{2}(1 - x_C). \end{aligned}$$

Then, voters with signal -1 ex ante prefer L to C if and only if $\gamma_{-1} \geq \gamma_{-1}^*$ and voters with signal $+1$ ex ante prefer R to C if and only if $\gamma_1 \geq \gamma_1^*$.

We compute the sincere/semi-sincere equilibria for this agenda in the Appendix (see Proposition 4 there). The following tables summarize the sincere/semi-sincere equilibria and their outcomes for agenda $[L, \{C, R\}]$, where the strategy profile in

quotation marks is semi-sincere, while all other strategy profiles are fully sincere:¹⁷

	$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C))$	$\gamma_1 \in (\frac{1}{2}(1+x_C), 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$	$(\neg L_1 C_2, \neg L_1 C_2)$	$"(\neg L_1 C_2, \neg L_1 C_2)"$	$(\neg L_1 C_2, \neg L_1 \neg C_2)$
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$

Table 2: Sincere/Semi-sincere equilibria for agenda $[L, \{C, R\}]$

The possible outcomes under agenda $[L, \{C, R\}]$ are:

		$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1+x_C}{2})$	$\gamma_1 \in (\frac{1+x_C}{2}, 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \frac{1-x_C}{2})$	$n_{-1} \geq n+1$	C	C	C
	$n_{-1} \leq n$	C	C	R
$\gamma_{-1} \in (\frac{1-x_C}{2}, \gamma_{-1}^*)$	$n_{-1} \geq n+1$	C	C	C
	$n_{-1} \leq n$	C	C	R
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$n_{-1} \geq n+1$	L	L	L
	$n_{-1} \leq n$	R if $n_{-1} < [k]$ C otherwise	R if $n_{-1} < [k]$ C otherwise	R

Table 3: Equilibrium outcomes for agenda $[L, \{C, R\}]$

The results for agenda $[R, \{C, L\}]$ are analogous. When voters with signal -1 are in ex ante minority, agenda $[L, \{C, R\}]$ that puts the ex ante more extreme alternative L up for vote first dominates $[R, \{C, L\}]$ from the Condorcet perspective when γ_1 is relatively small: alternative R may be hastily rejected in the first stage under $[R, \{C, L\}]$ even though R rather than C may be the Condorcet winner. This happens under $[R, \{C, L\}]$ because these voters do not have information about n_{-1} to properly compare R and C when R is put up for vote in the first stage. If γ_{-1} is not too small, they do have enough information under agenda $[L, \{C, R\}]$ when C is put up for vote in the second stage.

¹⁷We take (half-)open intervals to exclude the cutoff points $\gamma_1^*, \gamma_{-1}^*, (1+x_C)/2$ and $(1-x_C)/2$ so that the respective sincere/semi-sincere equilibrium is unique.

5.1 Electable Compromises

Equipped with the above equilibria, we first prove a key Lemma that identifies the different compromise locations effectively leading to its election. For a given n and for a given realization of signals, let $x_C^L(n_{+1})$ denote the compromise location such that, ex-post, voters with signal -1 are indifferent between L and C :

$$\frac{-1 + x_C^L(n_{+1})}{2} = -\gamma_{-1} + (1 - \gamma_{-1}) \frac{1}{2n} (n_{+1} - (2n - n_{+1})),$$

which yields

$$x_C^L(n_{+1}) = 2(1 - \gamma_{-1}) \frac{n_{+1}}{n} - 1.$$

For a given n , voters with signal -1 are ex-ante indifferent between L and C if the compromise is located at

$$x_C^L = 4(1 - \gamma_{-1})(1 - p) - 1.$$

For voters with signals $+1$ we analogously define:

$$x_C^R(n_{-1}) = 1 - 2(1 - \gamma_1) \frac{n_{-1}}{n}, \quad \text{and} \quad x_C^R = 1 - 4(1 - \gamma_1)p.$$

Lemma 1 *Consider agenda $[L, \{C, R\}]$, and suppose that Assumption A holds.*

- a. *If $n_{-1} \geq n + 1$, then compromise C is elected if and only if $x_C \leq x_C^L$. Otherwise, alternative L is elected.*
- b. *If $n_{-1} \leq n$, then compromise C is elected if and only if $x_C \in [-1 + 2\gamma_1, x_C^L] \cup [x_C^R(n_{-1}), 1]$. Otherwise, alternative R is elected.*

If voters with signal -1 form an ex post majority, C will be elected if it is close enough to L so that even those voters find it more attractive than L , i.e., if $x_C \leq x_C^L$. If voters with signal $+1$ form an ex post majority, C will be elected if $x_C \geq x_C^R(n_{-1})$, but it can also get elected even if $x_C < x_C^R(n_{-1})$. In particular, C is elected if $-1 + 2\gamma_1 \leq x_C \leq \min\{x_C^L, x_C^R(n_{-1})\}$. In this case, the equilibrium profile is $(-L_1C_2, -L_1C_2)$, and C is elected with unanimous support because no information is released at the first stage.

5.2 Optimal Compromise Location

In this Subsection we assume that the agenda setter has a single-peaked utility function that is maximized at some alternative $x^* \in [-1, 1]$, and that it locates the compromise in order to maximize the expected utility it derives from the elected alternative. For example, the empirical analysis of Martin and Vanberg [2014] suggests that the most likely compromise within coalition governments is an average of the positions of the represented parties.

1) We first assume that $x^* \in (-1, 1)$. The agenda setter proposes here a compromise policy because her first-best alternative is not a priori on the table. For example, x^* could be the policy position that maximizes the expected utility of the members of the majority party, or the expected utility of all voters, etc. The key observation is that the location must be chosen such that the compromise is elected with high probability. Therefore, the agenda setter should set $x_C = x^*$ if x^* belongs to the set of electable compromises (characterized in Lemma 1), and otherwise choose among the electable compromise locations the one that is closest to x^* . Formally, we have:

Proposition 2 *Assume that $\max\{\gamma_{-1}, \gamma_1\} < 1$ and that the agenda setter has a utility function that is symmetric around its peak $x^* \in (-1, 1)$. Suppose that Assumption A holds, and that $-1 + 2\gamma_1 \neq x_C^L$.*

Let

$$X_C \equiv [-1 + 2\gamma_1, x_C^L] \cup [x_C^R, 1]$$

be the set of compromise locations that get elected with agenda $[L, \{C, R\}]$ and suppose that $\min_{x \in X_C} |x - x^|$ has a unique solution. Then, the optimal compromise location $x_C(n)$ satisfies*

$$\lim_{n \rightarrow \infty} x_C(n) = \arg \min_{x \in X_C} |x - x^*|.$$

2) Suppose now that $x^* = 1$. That is, the goal of the agenda setter is to elect an alternative that is as close as possible to R .¹⁸ Since R is already on the table, the

¹⁸This may be the case, for example, if its constituent base supports that position more strongly than the legislators themselves.

only reason for the agenda setter to propose a compromise in this case is to prevent alternative L from being elected. Thus, the compromise could be seen as a form of "killer amendment" (see for example [Riker 1986]). From the agenda setter's point of view, the compromise would ideally be elected if voters with signal -1 have a majority (it prevents then outcome L), but not be elected if voters with signal $+1$ have a majority (so that their ideal policy R gets elected then). The location of the compromise must therefore take into account the precise conditions under which the compromise will be elected.

Proposition 3 *Suppose that the agenda setter chooses the location of the compromise $x_C(n)$ in order to maximize the expected location of the elected alternative, i.e., $x^* = 1$. Consider agenda $[L, \{C, R\}]$ and assume that Assumption A holds. The optimal compromise C is located at*

$$x_C(n) = \begin{cases} \text{just below } -1 + 2\gamma_1 \text{ or at } x_C^L & \text{if } x_C^L \geq -1 + 2\gamma_1 \\ \text{at } x_C^L & \text{if } x_C^L < -1 + 2\gamma_1 \end{cases}.$$

The optimal compromise location for a "traditionalist" agenda setter is often determined by the position $x_C = x_C^L$ that makes voters with the opposite signal ex ante indifferent between the compromise and their own traditional position: this is the highest compromise that will still be elected if those voters do have a majority. Sometimes it is even better to choose a location further to the left: rather than appealing to voters with the opposite signal, such a move makes the compromise less attractive for voters with signal $+1$ and increases the chance that R will be elected when they have a majority.

6 The Outcome of Voting on the Flag

We now come back to our case study and make a brief consistency check between the above theoretical considerations and the observed outcome.

Note first that, consistent with our model, the agenda was indeed convex and the government's proposal BRG – that presumably had the highest ex ante support – was

put to vote last. The following table displays in disaggregated form the results of the two votes, first on BWR and then on BRG/BWR:¹⁹

	$Y - Y$	$Y - N$	$N - N$	$N - Y$
SPD	0	0	0	106
Z	10	1	0	49
DDP	21	19	0	14
DNVP	0	33	0	0
USPD	0	0	18	0
DVP	1	16	0	0
BBB	0	0	0	1
DHP	0	0	1	1
BVP	0	0	0	1
Total	32	69	19	172

Table 4: The disaggregated voting outcome

1. 106 out of the 107 present SPD members voted N-Y.²⁰ This is consistent with either a peak on the compromise BRG/BWR, or with an initial peak on BRG together with a subsequent shift to BRG/BWR caused by the relatively large and vocal support for BWR.²¹ Fixing the behavior of all other actors, 106 No votes of SPD members would have led to a clear rejection of the compromise and to the election of the Government’s proposal BRG. The omission to do so therefore suggests that these voters shifted their vote from BRG to the compromise BRG/BWR. The interdependent component of their preferences was clearly

¹⁹111 members missed both votes, most of them from the ruling coalition. For simplicity, we do not list here the few incomplete profiles where one roll-call was missed, nor the few profiles that contained abstentions. Adding all these, a total of 17 (out of which 8 voted Yes on the compromise), does not change the result or its interpretation.

²⁰One member missed the first vote, and voted Y on the second.

²¹Note that, given the chosen agenda, this shift is immaterial for the behavior in the first two votes on Red and BWR.

expressed in the Government's willingness to adjust its initial proposal to allow for a later determination of a fleet flag. Hence, we conclude that, after observing more than 100 votes in favor of BWR—about one third of the total—the members of the SPD most likely shifted their votes from an initial peak BRG to the compromise BRG/BWR.

2. All 18 present members of USPD voted N-N, consistent with sincere voting given a presumed peak on the left-most alternative Red. Their preference over Red is presumably very strong so that their interdependent component is not enough for them to shift their vote.
3. A large majority of members of the centrist party Zentrum voted N-Y (49), while 7 abstained/missed the first vote and voted Y at the second. This is, again, consistent with either a peak on BRG/BWR, or with a peak on BRG and a subsequent shift after the BWR vote. The fact that the compromise BRG/BWR was formally proposed by this party, points to the first alternative. 10 other members of Zentrum voted Y-Y, which is consistent with sincere voting and a peak on BWR.
4. The other centrist party, DDP, was also split: 21 members voted Y-Y, consistent with a peak on BWR,²² while 14 of its members voted N-Y, which, as we discussed above, is consistent with a peak on the compromise BRG/BWR.
5. 49 out of the 50 present members of the right-wing, conservative parties DNVP and DVP, voted Y-N.²³ They were joined in this voting pattern by 20 members of the coalition (19 DDP and 1 Zentrum). All these voters had a presumed peak on BWR and were expected to vote Y-Y, since BRG/BWR was their remaining preferred alternative after the defeat of BWR. But they didn't, and their choice of seemingly dominated action cannot be explained by looking at the voting game in isolation. Therefore, we advance here a more speculative explanation: the radical conservative voters wanted to signal their unwillingness to compromise on the

²²Another 2 members of this party voted Y-Miss.

²³The remaining member voted Y and then missed the second vote.

flag, thus lending credibility to their threat of rejecting the entire constitution because of it. This explanation is a twist on Fenno’s “home-style” hypothesis.²⁴ Home-style—the need to justify behavior to constituents—is invoked to explain seemingly sub-optimal behavior (such as sincere voting) instead of behavior that exploits each strategic opportunity. Here sincere voting was in fact optimal but, at the last binary vote, it delivered the wrong signal. It is also very likely that the conservatives anticipated the vote shifting by the SPD, and hence believed that the compromise will be adopted anyhow, rendering their otherwise pretty risky signaling behavior costless.

To get an idea about the implied degree of interdependence, consider the agenda $[R, \{C, L\}]$ and the profile $(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$, which is an equilibrium if γ_1 is relatively high, i.e., when right-leaning voters are close to weighing solely their own signal. This assumption certainly fits well to their total unwillingness to compromise in the case study. We can then obtain an estimate of γ_{-1} , the weight on own signal of the left leaning members.

First, for the shifting parameter, we obtain that $\kappa = \frac{n}{2} \frac{1+x_C}{1-\gamma_{-1}} \leq 111$ (since 111 voters voted in favor of BGR). Observing that $n \approx 159$ (since about 319 voters participated in the vote), this yields $\gamma_{-1} \leq 0.3 - 0.7x_C$.

The compromise location x_C of the cantoned flag BRG/BWR is best thought to satisfy $-1 < x_C \leq 0$: it was definitely closer in spirit to the left alternative BRG (main flag) than to right alternative BWR (canton). The weighted average compromise between the parties forming the government coalition (in the spirit of the analysis of Martin and Vanberg [2014]) would be about $x_C = -0.52$ in this case. Setting $x_C = -0.5$ yields $\gamma_{-1} \leq 0.65$, a high degree of interdependence that can explain the governing coalition’s willingness to compromise.

Finally, the compromise flag BRG/BWR was the likely Condorcet winner since

$$n + 1 = 160 < n_{-1} \simeq 201 < 208 \leq 2n + 1 - \lceil \kappa \rceil,$$

where we estimated n_{-1} by counting the legislators that voted against BWR.

²⁴See [Fenno 1978], [Denzau and Riker and Shepsle 1985], and [Austen-Smith 1992].

Appendix A: The Hoeffding Inequality

For the proof of Proposition 2 we need to consider the case where the number of voters is large and we employ a well-known probabilistic tool, the Hoeffding Inequality. The implied approximation is very precise for large democracies - those often have more than 500 members of parliament.²⁵

Definition 2 *A random variable X is σ -subgaussian if for all $t \in \mathbb{R}$ there is $\sigma > 0$ such that its moment generation function $E(e^{tX})$ satisfies $E[e^{t(X - \mathbb{E}[X])}] \leq e^{\sigma^2 t^2 / 2}$.*

A Bernoulli random variable $X \sim \text{Bernoulli}(p)$ is σ -subgaussian with $\sigma = 1/2$. A binomial random variable $X \sim B(N, p)$, the sum of N independent Bernoulli random variables, is $\sqrt{N}/2$ -subgaussian. Any σ -subgaussian random variable X satisfies the Hoeffding bounds: for all $t \geq 0$,

$$\Pr \{X - \mathbb{E}[X] \geq t\} \leq e^{-t^2/(2\sigma^2)}, \quad (6)$$

and

$$\Pr \{X - \mathbb{E}[X] \leq -t\} \leq e^{-t^2/(2\sigma^2)}. \quad (7)$$

We shall use (6) or (7) to bound (tail) probabilities since the random variable \tilde{n}_{-1} , the number of agents with signal -1 , is $(\sqrt{2n+1})/2$ -subgaussian. For example, applying inequality (6) to \tilde{n}_{-1} yields

$$\Pr \{\tilde{n}_{-1} - \mathbb{E}[\tilde{n}_{-1}] \geq t\} \leq e^{-2t^2/(2n+1)}. \quad (8)$$

By setting $t = n + 1 - E[\tilde{n}_{-1}]$, we can rewrite the above inequality as

$$\Pr \{\tilde{n}_{-1} \geq n + 1\} \leq e^{-2(n+1 - \mathbb{E}[\tilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})(1-2p)^2}. \quad (9)$$

In other words, if voters with signal -1 are in an ex ante minority (i.e., $p < 1/2$), the probability that they are in an ex post majority (i.e., $n_{-1} \geq n + 1$) decays exponentially to zero as n grows.

²⁵To get an idea about the involved numbers, consider $n = 250$ which yields 501 voters. Assume that $p_{-1}^L = p_{-1}^R = 0.45$ which gives an expected value for the number of -1 signals of 225, a minority. The probability of nevertheless having a majority - at least 251 voters - with this signal is then less than 1% !

Appendix B: Proofs

Proof of Proposition 1

Proof. (i) We first show that the profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is an equilibrium. We only need to consider signal realizations such that an individual voter is pivotal at a given stage. Consider first voters with signal -1 . They play a best response by voting for C in the second stage because their ideal point is at most

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n} (2n) = 1 - 2\gamma_{-1} \leq x_C.$$

Voter i with signal -1 is pivotal in the first stage if (1) there are $\lceil k \rceil - 1$ other voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n other voters having signal of -1 (in this case, voter i is pivotal between L and C). The ideal point of voter i for the first case is

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n} (-(\lceil k \rceil - 1) + (2n - \lceil k \rceil + 1)) = -\gamma_{-1} + \frac{1 - \gamma_{-1}}{n} (n - \lceil k \rceil + 1).$$

The ideal point for the second case is $-\gamma_{-1}$. Therefore, in the first case, voter i with signal -1 will vote for L in the first stage if

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{n} (n - \lceil k \rceil + 1) \leq \frac{1}{2} (x_C + 1) \Leftrightarrow \gamma_{-1} \geq \frac{n + 2 - 2\lceil k \rceil - nx_C}{4n - 2\lceil k \rceil + 2}. \quad (10)$$

Since $-2\lceil k \rceil + 2 \leq 0$ and $\gamma_{-1} \geq \frac{1}{2} (1 - x_C)$ by assumption, we get

$$\gamma_{-1} \geq \frac{1 - x_C}{2} \geq \frac{n - nx_C - 2\lceil k \rceil + 2}{2n - 2\lceil k \rceil + 2} \geq \frac{n - 2\lceil k \rceil + 2 - nx_C}{4n - 2\lceil k \rceil + 2}.$$

Therefore, condition (10) is always satisfied. In the second case, since $\gamma_{-1} \geq \frac{1}{2} (1 - x_C)$, voter i with signal -1 will vote for L in the first stage.

Consider next voters with signal $+1$. By the definition of cutoff k , they play a best response in the second stage by voting for C if and only if at least $\lceil k \rceil$ voters support L in the first stage. Voter i with signal $+1$ is pivotal in the first stage if (1) there are $\lceil k \rceil - 1$ voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n voters having signal of -1 (in this case, voter i is pivotal

between L and C). Therefore, for the first case, voter i plays a best response by voting against L in the first stage if

$$\gamma_1 + \frac{1 - \gamma_1}{2n} (-(\lceil k \rceil - 1) + (2n - \lceil k \rceil + 1)) \geq \frac{1}{2}(1 + x_C),$$

which always holds since $\lceil k \rceil - 1 \leq k$ and k satisfies by definition

$$\gamma_1 + \frac{1 - \gamma_1}{2n} (-k + (2n - k)) = \frac{1}{2}(1 + x_C).$$

For the second case, since her ideal point $\gamma_1 \geq \frac{1}{2}(x_C - 1)$, voter i plays a best response by voting against L in the first stage.

To see that the complete information Condorcet winner will be selected under this strategy profile, note that. if at least $n + 1$ voters have signal -1 , L will be selected and this is the preferred alternative for voters with signal -1 since $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$. If at least $n + 1$ voters have signal $+1$, L will be rejected in the first stage. In the second stage, agents are, essentially, completely informed about signals of others, and C gets elected if and only if voters with signal $+1$ prefer C to R given the realized preferences.

Finally, note that if $k > n$, then, whenever L receives at least $\lceil k \rceil$ votes in the first stage, L is chosen and thus vote shifting from voters with signal $+1$ never occurs in equilibrium. In order for vote shifting to possibly occur in equilibrium, we must have $k \leq n$, which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$.

(ii) To obtain a contradiction, suppose there is a pure strategy profile that always selects the Condorcet winner, and let σ denote the corresponding profile of actions for the first stage. For each voter i this yields a mapping $\sigma_i : \{-1, +1\} \rightarrow \{L, \neg L\}$. Observe that, for any signal realization such that $n_{-1} = n + 1$, L is not the Condorcet winner: the ideal point of voters with signal -1 is then $-\gamma_{-1}$, which is closer to x_C than to -1 because $\gamma_{-1} < \frac{1}{2}(1 - x_C)$.

Suppose first that there are at most n voters whose strategy satisfies $\sigma_i(-1) = \neg L$. Consider a signal realization such that these voters have signal $+1$, and such that $n_{-1} = n + 1$. Then L gets selected even though it is not the Condorcet winner because all voters that have signal -1 vote for L . We conclude that, for at least $n + 1$ voters, the

profile σ must satisfy $\sigma_i(-1) = \neg L$. But this implies that L will not be selected even if all voters have signal -1 , in which case L is the Condorcet winner, a contradiction.

Finally, since there is no pure strategy profile that always selects the Condorcet winner, there can also be no mixed strategy profile that always selects the Condorcet winner. ■

Proposition 4 Consider agenda $[L, \{C, R\}]$ and suppose that Assumption A holds.

(i) The profile $(\neg L_1 C_2, \neg L_1 C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma_{-1}^*$ and $\gamma_1 \neq \gamma_1^*$.

(ii) The profile $(\neg L_1 C_2, \neg L_1 \neg C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \frac{1}{2}(1 + x_C)$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma_{-1}^*$.

(iii) The profile $(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma_{-1}^*$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma_{-1}^*$, and such that k is not an integer.

(iv) There is no sincere equilibrium if $\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$ and $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C))$. The profile $(\neg L_1 C_2, \neg L_1 C_2)$ forms a semi-sincere equilibrium: no voter is ever pivotal, and voters with signal -1 use a sincere strategy.

Proof. Sincerity Considerations: Consider the first stage voting. By Assumption A, voters with signal -1 weakly prefer L to R ex ante. These voters prefer C to L ex ante if

$$-\gamma_{-1} + (1 - \gamma_{-1})(1 - 2p) \geq \frac{1}{2}(-1 + x_C)$$

which is equivalent to $\gamma_{-1} \leq \gamma_{-1}^*$. Voters with signal $+1$ prefer R to L ex ante if $\gamma_1 + (1 - \gamma_1)(1 - 2p) \geq 0$, which is always satisfied. Therefore, it is a sincere strategy for voters with signal $+1$ to always vote against L , and it is a sincere strategy for voters with signal -1 to vote for L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$.

For the second stage, we need to consider two possible cases:

(a) $\gamma_{-1} \leq \gamma_{-1}^*$: all voters vote against L in the first stage, and no information about n_{-1} is revealed. By Assumption A, voters with signal -1 prefer L to R ex ante, and

because $\gamma_{-1} \leq \gamma_{-1}^*$ they also ex ante prefer C to L . As a result, these voters prefer C to R ex ante. Voters with signal $+1$ prefer R to C ex ante if

$$\gamma_1 + (1 - \gamma_1)(1 - 2p) \geq \frac{1}{2}(1 + x_C)$$

which is equivalent to $\gamma_1 \geq \gamma_1^*$. Therefore, if $\gamma_{-1} \leq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C , while for voters with signal $+1$ it is sincere to vote against C if and only if $\gamma_1 \geq \gamma_1^*$.

(b) $\gamma_{-1} \geq \gamma_{-1}^*$: not all voters vote against L and n_{-1} is revealed. In this case, voters with signal -1 always prefer C to R , because $\gamma_{-1}^* \geq \frac{1}{4}(1 - x_C)$ and because a voter with signal -1 prefers C to R in the situation where she is the lone -1 voter if

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(2n) \leq \frac{1}{2}(1 + x_C) \Leftrightarrow \gamma_{-1} \geq \frac{1}{4}(1 - x_C).$$

By the definition of the cutoff k , voters with signal $+1$ prefer R to C in the second stage if and only if $n_{-1} \geq k$. Therefore, if $\gamma_{-1} \geq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C , while voters with signal $+1$ vote sincerely against C if and only if $n_{-1} \geq k$.

To summarize, in the first stage, it is a sincere strategy for voters with signal -1 to vote in favor of L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$, and for voters with signal $+1$ to always vote against L . In the second stage, sincerity requires that (1) if $\gamma_{-1} \leq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $\gamma_1 \leq \gamma_1^*$ while voters with signal -1 always vote for C , and that (2) if $\gamma_{-1} \geq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $n_{-1} \geq k$ while voters with signal -1 always vote for C . As a result, the profiles $(\neg L_1 C_2, \neg L_1 C_2)$, $(\neg L_1 C_2, \neg L_1 \neg C_2)$, and $(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$ are the unique sincere profiles corresponding to the cases $(\gamma_{-1} \leq \gamma_{-1}^* \text{ and } \gamma_1 \leq \gamma_1^*)$, $(\gamma_{-1} \leq \gamma_{-1}^* \text{ and } \gamma_1 \geq \gamma_1^*)$, and $(\gamma_{-1} \geq \gamma_{-1}^*)$, respectively. Moreover, if k is not an integer, and if $\gamma_{-1} \neq \gamma_{-1}^*$, and $\gamma_1 \neq \gamma_1^*$ the sincere strategies are uniquely defined.

Equilibrium Considerations: In profiles $(\neg L_1 C_2, \neg L_1 C_2)$ and $(\neg L_1 C_2, \neg L_1 \neg C_2)$, all voters vote against L in the first stage, so that we only need to consider pivotality at the second stage. Conditional being on pivotal between R and C , voters with signal $+1$ prefer C if and only if $\gamma_1 \leq \frac{1}{2}(1 + x_C)$, while voters with signal -1 prefer C for

all γ_{-1} . It follows from the proof of Proposition 1 that profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is an equilibrium if $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$. Note that, by Assumption A we obtain $\gamma_1^* \leq \frac{1}{2}(1 + x_C)$ and $\gamma_{-1}^* \geq \frac{1}{2}(1 - x_C)$. Therefore, $(\neg L_1C_2, \neg L_1C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*$, $(\neg L_1C_2, \neg L_1\neg C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \frac{1}{2}(1 + x_C)$, and $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma_{-1}^*$.

Finally, when $\gamma_{-1} < \gamma_{-1}^*$ and $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C)]$, the non-existence of sincere equilibrium is due to a conflict between sincerity and pivotality. Sincerity requires that both type of voters vote against L at the first stage and voters with signal -1 vote for C at the second stage. Therefore, no new information would be revealed by the vote at the first stage. If $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C)]$, then, based on ex ante information, voters with signal $+1$ prefer R to C . But, conditional on pivotality, this preference is reversed. Hence, sincere voting suggests that voters with signal $+1$ vote against C , but pivotality requires that they vote in favor of C . Thus, sincere voting by voters with signal $+1$ cannot be part of an equilibrium in this case. ■

Proof of Lemma 1

Proof. Suppose that $n_{-1} \geq n + 1$. Then C gets elected if and only if $\gamma_{-1} \leq \gamma_{-1}^*$ (see Table 3). By definition, -1 voters are ex ante indifferent between L and C if $\gamma_{-1} = \gamma_{-1}^*$ or if $x_C = x_C^L$. Therefore, $\gamma_{-1} \leq \gamma_{-1}^*$ is equivalent to $x_C \leq x_C^L$. Suppose next that $n_{-1} \leq n$. Then C gets elected either if (1) $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \frac{1+x_C}{2}$ or if (2) $\gamma_{-1} \geq \gamma_{-1}^*$, $\gamma_1 \leq \frac{1+x_C}{2}$ and $n_{-1} \geq k$ (see Table 3). As shown above, $\gamma_{-1} \leq \gamma_{-1}^*$ is equivalent to $x_C \leq x_C^L$. Also, $\gamma_1 \leq \frac{1+x_C}{2}$ is equivalent to $x_C \geq 2\gamma_1 - 1$. Hence, case (1) applies if and only if $2\gamma_1 - 1 \leq x_C \leq x_C^L$. Case (2) applies if and only if $x_C \geq x_C^L$, $x_C \geq 2\gamma_1 - 1$, and $n_{-1} \geq k$ (which is equivalent to $x_C \geq x_C^R(n_{-1}) = 1 - 2(1 - \gamma_1)\frac{n-1}{n}$). Since $n_{-1} \leq n$, we have $2\gamma_1 - 1 \leq x_C^R(n_{-1})$, and therefore, alternative C is chosen in case (2) if $x_C \geq \max\{x_C^L, x_C^R(n_{-1})\}$. Finally, since $n_{-1} \leq n$, we have $x_C^R(n_{-1}) \geq 2\gamma_1 - 1$, and thus the set of implementable compromise locations

$$[-1 + 2\gamma_1, x_C^L] \cup [\max\{x_C^L, x_C^R(n_{-1})\}, 1]$$

can be rewritten as

$$[-1 + 2\gamma_1, x_C^L] \cup [x_C^R(n_{-1}), 1].$$

■

Proof of Proposition 2

Proof. For any x in the interior of X_C , the probability that x gets elected converges to 1. To see this, note that, for any $t > 0$, it follows from (7) that

$$\begin{aligned} \Pr \{x_C^R(\tilde{n}_{-1}) - x_C^R \geq t\} &= \Pr \left\{ 2(1 - \gamma_1) \left(2p - \frac{\tilde{n}_{-1}}{n} \right) \geq t \right\} \\ &= \Pr \left\{ \frac{\tilde{n}_{-1}}{n} - 2p \leq -\frac{t}{2(1 - \gamma_1)} \right\} \leq e^{-\frac{2t^2}{4(2n+1)(1-\gamma_1)^2}}. \end{aligned}$$

Therefore, any compromise $x > x_C^R$ is elected with probability approaching 1 as n grows. By analogous arguments, any compromise $x \in (-1 + 2\gamma_1, x_C^L)$ is elected with probability approaching 1. For $x \notin X_C$, however, the probability of x being elected converges to 0. We assume by contradiction that $\lim_{n \rightarrow \infty} x_C(n) \neq \arg \min_{x \in X_C} |x - x^*|$. Suppose first that $x^* \in X_C$, but there exists $\varepsilon > 0$ such that $x_C(n) > x^* + \varepsilon$ for infinitely many n (the argument is analogous if $x_C(n) < x^* - \varepsilon$ for infinitely many n). Then, there exists $x \in \text{int}(X_C)$ that is sufficiently close to x^* such that the utility of the agenda setter if x gets elected is strictly higher than those obtained when $x^* + \varepsilon$ is elected or R are elected. Since the probability that x gets elected converges to 1, and since the agenda setter's utility is single-peaked, we conclude that, for n large enough, it is strictly better to propose x than to propose any compromise above $x^* + \varepsilon$. Since $x_C(n)$ is optimal by assumption, this yields a contradiction.

Suppose now that $x^* \notin X_C$ and that $\arg \min_{x \in X_C} |x - x^*|$ is a singleton, denoted by x' . To obtain a contradiction, suppose $x_C(n) > x' + \varepsilon$ or $x_C(n) < x' - \varepsilon$ for infinitely many n , and let $x \in \text{int}(X_C)$ and sufficiently close to x' . Then x gets elected with probability approaching 1 and, conditional on being elected, provides strictly greater utility compared to $x' + \varepsilon$ and compared to R . It follows that, for n large enough, it is strictly better to propose compromise x than to propose compromise $x_C(n)$, a contradiction. ■

Proof of Proposition 3. Note that Assumption A implies $x_C^L \leq 1$. Also, observe that $n_{-1} \leq n$ implies $-1 + 2\gamma_1 \leq x_C^R(n_{-1})$.

(a). If $-1 + 2\gamma_1 \leq x_C^L$, then, by Lemma 1, we can set x_C just below $-1 + 2\gamma_1$ to get C elected if $n_{-1} \geq n + 1$, and get R elected if $n_{-1} < n + 1$. On the other hand, setting $x_C = x_C^L$ will always get C elected. No other compromise location can be optimal: Setting x_C substantially below $-1 + 2\gamma_1$ is dominated by setting it just below $-1 + 2\gamma_1$. Setting x_C between $-1 + 2\gamma_1$ and x_C^L is dominated by setting $x_C = x_C^L$, while setting x_C above x_C^L is dominated by setting it just below $-1 + 2\gamma_1$.²⁶

(b). If $x_C^L < -1 + 2\gamma_1$, then Lemma 1 implies that if we set $x_C = x_C^L$, then if $n_{-1} \geq n + 1$ alternative C gets elected, while if $n_{-1} < n + 1$ alternative R gets elected. Any higher compromise location is worse because such a compromise will never be elected if $n_{-1} \geq n + 1$. Any lower compromise location is worse because it will be elected in the same instances, but will provide lower utility conditional on being elected. ■

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²⁶Setting x_C just below $2\gamma_1 - 1$ is not dominated by $x_C = x_C^L$, because R is elected when x_C is set just below $2\gamma_1 - 1$ and $n_{-1} \geq n + 1$. Hence, the optimal location depends on the probability of a realized majority of voters with signal -1 . It is optimal to locate the compromise at x_C^L if

$$x_C^L \geq \Pr(\tilde{n}_{-1} \geq n + 1) \cdot (2\gamma_1 - 1) + \Pr(\tilde{n}_{-1} \leq n) \cdot 1.$$

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