

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{N(N-1)}{n(n-1)} y_i y_j + \sum_{i=1}^n \frac{N-1}{n} \binom{N}{n} \binom{n}{1} y_i^2$$

$$= \sum_{i=1}^n \left\{ \frac{N}{n} \left(1 - \frac{N-1}{n-1}\right) y_i^2 + \frac{N(N-1)}{n(n-1)} \right\} y_i$$

$$+ \sum_{i=1}^n \sum_{j=1}^n \frac{N(N-1)}{n(n-1)} y_i y_j$$

$$= \binom{n}{1} \left\{ \frac{N}{n} \sum_{i=1}^n y_i^2 - \frac{N(N-1)}{n(n-1)} \sum_{i=1}^n y_i^2 \right\}$$

$$\textcircled{\text{I}} = \binom{n}{1} \sum_{i=1}^n \frac{N}{n} y_i^2 + \binom{n}{1} \sum_{i=1}^n \frac{N(N-1)}{n(n-1)} y_i^2$$

$$\textcircled{\text{II}} = \binom{n}{1} \sum_{i=1}^n \pi_i y_i \left(\sum_{j=1}^n \pi_j y_j \right) = \sum_{i=1}^n \pi_i^2 y_i^2$$

$$= \textcircled{\text{II}} - \textcircled{\text{I}}$$

$$= \binom{n}{1} \sum_{i=1}^n \sum_{j=1}^n \pi_i \pi_j y_i y_j - \binom{n}{1} \sum_{i=1}^n \sum_{j=1}^n \pi_i \pi_j y_i y_j$$

$$= \binom{n}{1} \sum_{i=1}^n \sum_{j=1}^n y_i y_j (\pi_i \pi_j - \pi_i \pi_j)$$

$$= \binom{n}{1} \sum_{i=1}^n \sum_{j=1}^n y_i y_j \text{Cov}(I_i, I_j)$$

$$= \text{Cov} \left(\sum_{i=1}^n I_i y_i, \sum_{j=1}^n I_j y_j \right)$$

$$\text{Var} \left(\sum_{i=1}^n y_i \right) = \text{Cov} \left(\sum_{i=1}^n y_i, \sum_{j=1}^n y_j \right)$$

$$1 - f = f - 1$$

$$N \gg u \quad f = 0 \\ N = u \quad f = 1$$

$$f = \frac{N}{u}$$

$$S^2 = \frac{1}{T} \sum (y_i - \bar{y})^2$$

$$S^2(f) = S^2 \left(1 - \frac{N}{u}\right) \left(\frac{u}{T}\right) =$$

$$\left[\sum (y_i - \bar{y})^2 \right] \left(\frac{N}{T}\right) \left(\frac{1-N}{u-N}\right) \left(\frac{u}{T}\right) =$$

$$\left[\sum N \frac{1-N}{u-N} - \sum \frac{1-N}{u-N} \right] \left(\frac{N}{T}\right) \left(\frac{u}{T}\right) = \langle \sigma^2 \rangle_{\text{var}}$$

Thus

$$\left[(u-N)N - \right] \frac{1-N}{u} =$$

$$\left[Nu + N^2 - N^2 \right] \frac{1-N}{u} =$$

$$\left[Nu + \cancel{Nu} - N^2 - \cancel{N^2} \right] \frac{1-N}{u} =$$

$$\left[\frac{1-N}{(1-N)Nu} - \frac{1-N}{(1-N)N} \right] \sum =$$

$$\sum Nu - \sum N \frac{1-N}{1-N}$$

$$\left[\sum Nu - \sum N \frac{1-N}{1-N} + \sum \frac{1-N}{u-N} \right] \left(\frac{N}{T}\right) \left(\frac{u}{T}\right) =$$

$$\sum - \sum N \frac{1-N}{u-N} \frac{N}{u} \left(\frac{u}{T}\right) +$$

$$\sum \frac{1-N}{u-N} \frac{N}{u} \left(\frac{u}{T}\right) = \langle \sigma^2 \rangle_{\text{var}}$$