We want to show

$$\sum_{i=1}^{N} \tilde{\xi}_{Ni} \xrightarrow{d} N(0,1).$$

This will follow from the Lindeberg CLT if we can show the Lindeberg condition

$$\sum_{i=1}^{N} E\left[\tilde{\xi}_{Ni}^{2} I(|\tilde{\xi}_{Ni}| > \varepsilon) \right] \to 0 \qquad \forall \varepsilon > 0.$$

Now,

$$LHS = \sum_{i=1}^{N} E\left[\frac{\gamma_{Ni}^2}{\tau^2} \xi_i^2 I\left(|\xi_i| > \varepsilon \frac{\tau}{\gamma_{Ni}}\right) \right]$$

$$= \sum_{i=1}^{N} \frac{\gamma_{Ni}^2}{\tau^2} E\left[\xi_i^2 I\left(|\xi_i| > \varepsilon \frac{\tau}{\gamma_N^*}\right)\right]$$

$$\gamma_N^* = \max_i \gamma_{Ni}$$

$$= E\left[\xi^2 I\left(|\xi| > \varepsilon \frac{\tau}{\gamma_N^*}\right)\right] \sum_{i=1}^{N} \frac{\gamma_{Ni}^2}{\tau^2}$$

$$= E\left[\xi^2 I\left(|\xi| > \varepsilon \frac{\tau}{\gamma_N^*}\right)\right] \to 0$$

since $\frac{\tau}{\gamma_N^*} \to \infty$.