Government Support of Investment Projects in the Private Sector: 
A Microeconomic Approach

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ABSTRACT

We examine government decisions on subsidizing investments in the private sector and discriminating among firms in its support programs. By taxing corporate profits, the government may affect corporate investment decisions, causing firms to invest less than what would be socially optimal. Investments that are desirable from the standpoint of social welfare may be rejected by shareholders, which may ultimately lead to the collection of fewer taxes. We analyze the conditions for optimal subsidies for investments carried out by the private sector. We find that high-risk ventures that generate substantial spillover activity are prime candidates for government incentive schemes.
Government Support of Investment Projects in the Private Sector:
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In this paper we conduct a theoretical analysis of government incentives for private investment activity. We focus on two major questions: First, under what conditions is a government justified, from an economic viewpoint, in supporting investment activities undertaken by private firms? Second, under which conditions is supporting an investment in R&D projects more economically justifiable than support of an investment in a traditional low-tech project?

In our analysis, we focus on the microeconomic interaction between the government and a specific pure-equity firm. Our approach is based on a partial equilibrium solution, within the framework of Modigliani and Miller’s (1958, 1963) (M&M) modeling of corporate valuation. We assume that there are no taxes but corporate taxes, and that within this framework all markets are in equilibrium. Following Modigliani and Miller (1958, 1963), we let governments "negotiate" with firms to maximize the total net value to all stakeholders.

We view the government as an economic agent that collects taxes from corporations based on simple, uniform tax rules, and which provides a basket of services to the entire population. Unlike the majority of academic papers in the public finance area, we assume that a government acts as an economic agent. Its objective is to enhance public welfare and in order to jointly achieve a higher level of public welfare, it “negotiates” with private firms. Therefore, the government takes into consideration the present value of its net tax collection from each new investment made in the private sector. Through its traditional tax rules, governments may affect and alter the investment decisions of the firm, causing a distortion in the allocation of resources throughout the economy. We examine the conditions under which the government may provide investment subsidies to an individual company to generate economic benefits and enhance the public welfare.

In this paper, we analyze the welfare effect of subsidies extended differentially to specific projects, rather than as an across-the-board tax break. We find that subsidizing industrial R&D is not necessarily inefficient and can benefit individual firms, industries and the economy as a whole.

We adopt a microeconomic approach, looking mainly at the firm’s level and modeling the decision-making process for corporate investments and how those investments affect social welfare. We do not presume to prove an optimal corporate tax schedule that eliminates or minimizes tax distortions. Rather, we take the tax regime as given: corporate taxes are levied at a uniformed rate on accounting profits.

Taxation affects the investment opportunity function in two fundamental ways. It reduces the NPV of the firm for any investment scope, and it reduces the
firm’s optimal level of investment relative to a no-tax scenario. Thus, we are able to identify three general cases in which mitigation of corporate taxation can be justified.

In the first case, corporate taxation may cause the firm to reject projects with a positive pre-tax NPV. By rejecting the project, the firm loses the incremental NPV generated and the government loses the tax. In these instances, the government may be justified in providing subsidies so that the project is accepted.

A second justification for subsidizing investments is found in cases in which we can identify positive externalities to a specific investment. A given private investment may positively impact its environment even though the individual firm is unable to capture the economic value of spillover effects. In this case, the government may intervene to generate spillovers to other sectors in the economy and to encourage entrepreneurs to increase investment to a socially optimal level.

A third reason for governments to forgo a portion of potential tax revenues is in order to attain a socially optimal level of investment and avoid under-investment. Our assumption is that at the socially optimal level of investment, the value added to the wealth of the entire society is maximized, benefiting both the firm and government. We define the social benefit of a project by its pre-tax NPV, (and, in the externality case, also by the addition to NPVs of other projects being affected by the specific project). With taxes, we measure the social benefit by the sum of the NPV of the project and the PV of taxes collected by the government.

This approach to the measurement of social welfare in an uncertain environment, where uncertain future social values are converted to present value terms, on an ex ante basis, differs from the analysis of social costs based on actual ex-post results.

In this paper we do not comment on the optimal subsidy or tax policy of the government. The starting point of our analysis is a discriminatory subsidy scheme in which the government provides grants for individual investment projects. Conventional flat-rate corporate taxes continue to be imposed on the firm’s accounting profits. The subsidy is conditional and requires that if the project is successful, the firm will repay the government royalties from future cash flow. We limit our analysis to a one-period framework. In this framework, we show that with a flat tax rate and fixed government budget, a policy of subsidizing high-tech firms rather than low-tech firms is rational. We base this contention on the observation that high-tech firms typically incorporate two major characteristics that, taken together, make such firms prime candidates for government-induced incentives:

- The risk-return profile of high-tech ventures leads to a more frequent occurrence of suboptimal project rejection.
• There is evidence that high-tech firms generate a greater spillover effect that impacts positively on activity in other sectors of the economy.

The paper goes as follows. In section one we discuss the background to our study and survey literature. In section two we present the basic model for investment decisions by an individual firm under various regimes: no tax case, tax case, and tax and subsidies case. In section three we provide justification for subsidizing projects in the private sector. In section four we analyze the effect of externalities and we show how a government can induce entrepreneurs to increase investments and thus enhance the spillover effect. In section five the decision to subsidize high-tech versus low-tech projects is discussed. In section six we conclude and discuss our main results.

I. Background

The effect of corporate taxes on the investment decisions of a firm is analyzed by many authors, for example, Stiglitz (1973), Asimakopulos and Burbidge (1975), Galai (1998), and Brealey, et al. (1998). Stiglitz (1976) shows how tax can be non-distortive, but he concludes: "True economic depreciation and immediate write-off of capital expenditures are both depreciation policies which are equitable and efficient … Neither, however, is generally used; actual depreciation allowances introduce elements of inefficiency as well as inequity."

Galai (1998) extends M&M Proposition III by internalizing government’s claim on the corporation as part of the firm’s capital structure. He shows the different conditions under which the firm’s expanded capital structure will not affect the firm’s value. Mayshar (1977) constructs an economic model and investigates the effect of government subsidies on the decision of a risky firm.

In most countries, the corporate tax rate is uniform for all corporations. As a result, governments often find it easier to introduce differential subsidy schemes, on a case-by-case basis, rather than to adjust the tax code. In many countries, e.g. Ireland and Israel, special government units are established to analyze all subsidy applications. Subsidies are granted on a case-by-case basis, according to general guidelines. An analysis of such schemes and their impact on firms can be found in Levy and Terleckyj (1983), Mansfield and Switzer (1984), and Wallsten (2000). An international survey of governmental support schemes for R&D is included in a
policy paper published by the Canadian Department of Finance and Revenue,\(^1\) with a special emphasis on the G-7 countries and Canada.

Mansfield (1986) shows empirically that for the United States, Canada, and Sweden, tax credits for R&D reduces government revenues by substantially more than the amount added to the entrepreneur. Mansfield concludes that tax credits for R&D activity are inefficient.

There is a lot of evidence in the literature on the existence of externalities. A number of empirical papers have tried to quantify the spillover effect of investment in R&D on local markets and internationally, see, for example Jaffe (1986), Bernstein and Nadiri (1989), Griliches (1992), Branstetter (1996), Nadiri and Kim (1996). Externalities can be affected by taxation through the investment decisions of the relevant firms. See, for example, Griliches (1979), (1992), and (1995), Jones (1995), Levin and Reiss (1988), Mairesse and Mohen (1995), and Park (1996). See also Hall’s (1992) review of the R&D tax policy during the eighties. The practice of subsidizing R&D projects is discussed in Butler and Mitchel (1998), Irwin and Klenow (1994), and Lichtenberg (1987).

The effect of externalities of investments in research intensive firms drew special attention. In the report of the Department of Finance and Revenues, Canada, it says: “The economic rationale for governments to assist R&D is that the benefits of R&D spillover, or extend beyond the performers themselves, to other firms and sectors of the economy and the value of these benefits is not fully appropriable by the R&D performer. These “spillover benefits” mean that, in the absence of government support, firms would perform less R&D than is desirable from the economy’s point of view. Markets fail to allocate an efficient or socially optimal quantity of resources to the performance of R&D.”

Our starting point is the underinvestment caused by corporate taxation, but other authors focus on the over-investment incentives caused by the limited liability feature of the corporation. In John, Senbet, and Sundaram (1994), taxes induce underinvestment, as they do in our paper. However, John, Senbet, and Sundaram use the underinvestment incentive to counteract the overinvestment incentives of corporate limited liability. They claim that the problem arises even in the face of all-equity corporations, due to the generic corporate limited liability. Like their paper, our proposed tax incentives/solutions are targeted to specific industries (i.e., sector specific).

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\(^1\) “Why and how governments support research and development”, December 1997. See http://www.fin.gc.ca
II. Investment Decisions by an Individual Firm

We assume a company considering an investment in a project of $I$ dollars at time 0. The company expects the project to generate a net cash flow of $K(I)$ dollars at time 1 with probability of $\pi$, and 0 dollars with probability of $1-\pi$. We assume that $\pi e^{-r}$ and $(1-\pi)e^{-r}$ are the Arrow-Debreu present prices of the good and bad states of nature, respectively.\(^2\)

The firm faces a distribution of expected revenues with decreasing marginal return as the level of investment increases. We model this by a function, specified below in Equation (1).

The corporate tax rate is $\tau$ and it is applied to the profit from the project $K(I)-I$. The firm must determine the optimal size of the investment. For simplicity, we assume that $\pi$ does not vary with investment size.

The NPV of the investment is given by:

$$\text{NPV} = -I + \{(K(I) - I)(1-\tau) + I\} \pi e^{-r}$$

$$= -I + (K(I)(1-\tau) + \tau I) \pi e^{-r}$$

where $r$ is the continuously compounded risk-free rate for one period.

If we assume the tax code is of the full-loss offset tax type, than Equation (1) should be changed to reflect the fact that the tax subsidy $\tau I$ is certain. Hence, the expression for NPV becomes

$$\text{NPV} = -I + K(I)(1-\tau)\pi e^{-r} + \tau I e^{-r} = -I(1-\tau e^{-r}) + K(I)(1-\tau)\pi e^{-r}$$

(2)

If NPV < 0, the firm should reject the project. Suppose that without taxes the firm would have accepted the same project, i.e.,

$$\text{NPV}(\tau=0) = -I + K(I) \pi e^{-r} > 0$$

(3)

From here on, we assume that NPV($\tau=0$) > 0, and that without taxes and subsidies the firm will opt to invest in the project. By imposing the tax, if NPV<0, the firm rejects the project, and therefore the government cannot receive tax revenues from it.

The optimal investment decision for the no tax case is $I^*$, such that

$$K'(I^*) = \frac{1}{\pi e^{-r}}$$

(4)

For the tax case, the optimal investment $I^{**}$ is achieved when:

\(^2\) One can also consider $\pi$ and $1-\pi$ as risk-neutral probabilities of the two states. These are risk-adjusted probabilities in that the value of an uncertain project is equal to the expected payoff under this probability, discounted at the risk free rate. See Harrison and Kreps (1979) for a discussion of risk-neutral probabilities.
Figure 1 depicts the optimal investments for the tax and no tax cases.

Insert Figure 1 here

The distortion caused by the existing corporate tax can be corrected by different policies. One approach is to impose corporate taxes on cash flows (similar in many respects to value added tax) rather than accounting profits. We focus on an alternative scheme that has been used to support corporate investments, especially in industrial R&D. This scheme provides government funding for industrial R&D that is either partially or fully repayable in the future, should the investment prove successful. Repayment can take several forms, such as royalties on gross revenues or profits. This strategy is similar to the government providing a conditional loan to the corporation.

We assume that the government is willing to incur a proportion $\alpha$ of the investment in the project. In return, the government requires a payment of proportion $\beta$ of $I$ at time 1 only if the project is successful. In Equation (6) we assume that the payback to the government is proportional to the initial investment $I$. An alternative procedure is based on the pay-back on the revenues $K(I)$ up to a ceiling, $cI$, which is a function of the original investment: $\beta \cdot K(I)$, for $\beta \cdot K(I) \leq cI$, and $cI$, otherwise.

We note that when $\beta = 0$, we deal with investment tax credit case. But when $\beta = \alpha$, if the project is a success then the firm returns the initial grant in nominal terms. In general, if $\alpha > 0$ and $\beta > 0$ it is similar to a government loan, where $\frac{\beta}{\alpha} - 1$ is the nominal interest and is paid only in a good state.

Given the above subsidy structure, the NPV of the projects, denoted by $NPV^S$, and is given by

$$NPV^S = -(1-\alpha)I + \{[(K(I) - \beta I - (1-\alpha)I)(1-\tau) + (1-\alpha)I]\} \pi e^{-\tau}$$

$$= -(1-\alpha)I + \{[K(I) - \beta I \cdot (1-\tau) + (1-\alpha)I \cdot \tau]\} \pi e^{-\tau}$$

We denote the optimal $I$ where $NPV^S$ reaches its maximum by $I^{**}$. We denote the amount by which the NPV in the tax case is less than 0 by $d$:

$$d(I) \equiv - NPV(I)$$

We denote the net government’s grant by $g$:

$$g(I) \equiv \alpha I - \beta (1-\tau) + \alpha I \cdot \pi e^{-\tau}$$

where $g$ denotes the PV of the effective subsidy, $\alpha I$ represents the investment grant, and the term in the brackets is the contingent repayment to the government net
When \( d > 0 \) and there are no subsidies, the investment is rejected. However, if for a given \( I \), \( d > 0 \) (i.e., the NPV is negative) and \( g > d \), then the investment is accepted.

### III. Why Should Government Subsidize Investments?

To give the firm an incentive to invest, the government must pay \( g \). In return, the government expects to collect corporate taxes and royalties, and to acquire other social benefits.

The government has the incentive to subsidize the marginal investment only if the direct net taxes it collects, minus the subsidy, is sufficiently large, assuming at this stage that the value of externalities is zero. The present value of the contingent tax collection, from the accepted project, is

\[
G(I) = \tau [K(I) - I] \pi e^{-r}
\]

In Equation (9), our assumption is that the same state prices apply to the government’s tax claim as to the firm’s present value.

The net present value of the government claim with subsidy is \( G^S \) defined by:

\[
G^S(I) = G(I) - g(I) = -\alpha I + \{ [K(I) - \beta I - (1-\alpha)I] \tau + \beta I \} \pi e^{-r}
\]

If, prior to the subsidy, \( d(I^{**}) > 0 \), the firm will reject the project even if it is socially desirable, i.e., \( \text{NPV}_{\tau=0}(I^*) > 0 \). However, if a subsidy scheme can be devised such that for \( I^{**} \), \( G > g > d \), where \( I^{**} \) is the optimal decision of the corporation given the tax and subsidy schemes, then from a purely economic standpoint the government is justified in subsidizing the investment.³ If at this point \( g \) is greater than \( d \), the company will accept the project. The second condition \( G > g \) states that at this point the government has net receipts from the accepted project.

We define \( H \), the "value added to the economy," which is derived from investing in a project. This value added to the economy is defined in present value terms as the sum of the present value accrued to the entrepreneur plus the present value of government receipts:

\[
H^S = \text{NPV}^S + G^S
\]

This term measures the total net value added to the economy by accepting the investment. It takes into account the risk of the projects, and also measures the negative and positive effects of taxation and subsidies on the willingness of

³ In Section IV we show that in the case of externalities it may be that even if \( G^S < 0 \) (i.e. \( G < g \)), the government may have an incentive to subsidize the investment.
shareholders to undertake the investment. Governments should remove obstacles that prevent the entrepreneur from accepting a project in which $H^S > 0$.

Equation (6) can be analyzed for the optimal investment decision for three basic cases:

1. There are no taxes and no subsidies ($\tau = \alpha = \beta = 0$)

2. There are corporate taxes but no subsidies ($\tau > 0$, $\alpha = \beta = 0$)

3. There are corporate taxes $\tau$, subsidies $\alpha$, and royalties $\beta$, respectively. Further, by assuming that $\alpha = 0$ and $\beta < 0$, we can create a contingent future government subsidy depending on the success of the project.

The partial derivative of (6) is

\[
\frac{\partial NPV^s}{\partial I} = -(1-\alpha) + (1-\tau)[K'(I) - \beta] + (1-\alpha)\tau e^{-r} = (1-\alpha)(\tau e^{-r} - 1) + (1-\tau)e^{-r}[K'(I) - \beta]
\]

By equating (12) to zero, we obtain the necessary condition for an optimal investment decision:

\[
K'(I) = \beta + \frac{(1-\tau)e^{-r}}{(1-\tau)e^{-r}}
\]

where $K'(I)$ is the derivative of $K$ with respect to $I$.

The government can reverse this equation and find pairs $\alpha$, $\beta$ that will lead the investor to the socially optimal level of $I = I^*$. Then we can describe $\beta$ as a function of $\alpha$ for the optimal $I = I^*$ as follows

\[
\beta = \frac{\alpha(1-\tau)e^{-r} - \tau(1-e^{-r})}{(1-\tau)e^{-r}}.
\]

Equation (13) can also be rewritten in the following way:

\[
K'(I) = \frac{(1-\tau)e^{-r}}{(1-\tau)e^{-r}} + \frac{\beta(1-\tau)e^{-r} - \alpha(1-\tau)e^{-r}}{(1-\tau)e^{-r}}
\]

The first term of Equation (15) is the traditional result of M&M Proposition on the cost of capital for the tax case.\(^4\) Equation (15) is an extension of the Proposition for the case of government subsidy, which is reflected in the second term.

Equation (15) implies that, for any given project, the investment grant can be fully offset by the contingent royalties, since only the linear combination of $\alpha$ and $\beta$ matters. Thus, as soon as an increase in $\alpha$ is accompanied by an appropriate increase in $\beta$, the optimal investment decision remains constant.

\(^4\) See for example Brealey and Myers (1996, ch. 19).
For each increase of the initial subsidy $\alpha$ by an amount $\Delta \alpha$, if the conditional royalty $\beta$ is increased by $\Delta \beta$ according to

$$\frac{\Delta \beta}{\Delta \alpha} = \frac{1-e^{-r\pi\tau}}{e^{-r}\pi(1-\tau)}$$

the NPV$^S$ function will not change. Hence, the optimal decision of the firm will remain intact. Equation (16) shows that the substitution between $\alpha$ and $\beta$ is risk-class specific, for all projects with the same probability, $\pi$.

We assume that the production function is exponential, belonging to the class of concave functions, as follows:

$$K(I) = c_1 - c_2 \cdot e^{\frac{I}{c_3}}$$  \hspace{1cm} (17)

where $c_1$, $c_2$, and $c_3$ are constant parameters. By substituting Equation (17) in Equations (6), (13), we express NPV as a function of $I$.

The optimal investment for the no-tax case, $I^*$, satisfies

$$\frac{c_2 e^{\frac{I^*}{c_3}}}{c_3} = \frac{1}{\pi e^{-r}}$$

(18)

An analytical expression can be derived when this necessary condition is satisfied. We provide a numerical example\(^5\) for the following parameters:

$$K(I) = 600 - 1200e^{\frac{I}{100}}, \quad \pi = 0.6, \quad r = 6\%$$

(19)

We find that the optimal investment is $191.4 with a positive NPV=$47.6. Hence, with no government intervention the firm will accept the project.

In Figure 2, the bold line describes the NPV$^S$ function compared to the NPV function for the no-tax case and for the tax-case. The NPV is depicted as a function of the investment $I$. As a result of taxes, the highest NPV is achieved at a lower level of investment than in the no-tax case. The investor will require a higher before-tax marginal return to compensate for the tax. In our numerical example, assuming a 40% tax rate ($\tau = 40\%$), maximum profit is reached at $I=$165.9, but, since the project yields a negative NPV(-2.4), it will be rejected. Thus both government and the firm forgo the opportunity for an added value.

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\(^5\) A detailed numerical example of the effect of taxes and subsidies will be provided by the authors upon request.
the optimality conditions with taxes and subsidies. With $\alpha=0.3$ and $\beta=0.2$, the optimal investment stands at $189.8$ and the NPV of the project is $NPV^S=26.7$. Therefore, the project should be accepted. We note that the subsidy rate assumed in the numerical example is in the range of subsidies provided in many countries including Ireland, Israel, Canada and more. The PV of government revenues is $G^S=20.9$. The value added to the economy $H^S$, is equal to $47.6 (=26.7+20.9)$.

This example shows how, in the case of a single company, the government can induce the firm to undertake a project profitable for both the company and the government by mitigating taxes with subsidies. Without the subsidies, this project does not benefit the society, since the firm rejects it. However, with a combination of taxes and subsidies, the firm and the government share both risk and profits.

IV. The Effect of Externalities

By introducing the effect of potential spillovers on the profitability of investments in the private sector, we can show that a government may have an additional incentive to subsidize projects, even if $G^S<0$ in the single-firm case. Usually, studies do not explicitly model externalities or spillover effects, since it is difficult to assess the impact of a firm’s investment on the output of other firms.

We incorporate externalities by extending our microeconomic analysis to a two-firm model. We assume that firm A faces a cash-flow function $K_A(I_A)$, which is a function of its investment $I_A$. In contrast, Firm B’s cash-flow function $K_B=K_B(I_A, I_B)$, is a function of its own investment $I_B$, as well as the investment of firm A, $I_A$. For example, A could be a high-tech firm developing new products and B, a provider of services to high-tech firms such as product design. Higher investment by A means higher expected cash flow for B.

To simplify our exposition and analysis without losing generality, we assume that $I_A$ affects $K_B$ but $I_B$ does not affect $K_A$. If this is the case, the tax imposed on A may affect the productivity of B and taxes collected from firm B. To adequately analyze the economic decision facing the government, we must take into account the possible spillovers between the activities of A on B.

Given the above assumptions, firm A faces the same decision criterion as before (see Equation (13)). Firm B faces the following problem

\[
\max_{I_B} NPV_B(I_B | I_A),
\]

where $NPV_B$ is an extension of Equation (6):

\[
NPV_B(I_B | I_A) = I_B(1-\alpha_B)(\tau_B\pi_B e^{-\tau} - 1) + (K_B(I_A, I_B) - \beta_B I_B)(1-\tau_B)\pi_B e^{-\tau}
\]

and $\tau_B$ is the tax rate of B, $\pi_B$ is the probability of B, and $\alpha_B$ and $\beta_B$ are the subsidy parameters for B. The spillover effect is embodied in the term $K_B(I_A, I_B)$.
In most cases, the corporate tax rate is the same for all companies (i.e., \( \tau_A = \tau_B \)). The trend worldwide is to deal with special situations on an industry or regional basis, not directly through the tax code, but indirectly through grants or subsidies.

If, due to taxes and assuming no subsidies, firm A decides to reject the investment \( I_A \), it may have a negative impact on the performance of B, and the government may lose tax revenues from both A and B. In addition, there may be a welfare loss due to firm A’s decision to reject a socially desirable investment (i.e., an investment that has positive NPV without taxes) and an additional potential welfare loss due to an underinvestment of firm B.

If investments \( I_A \) and \( I_B \) are accepted, the present value of the government’s claims from taxes net subsidies from firms A and B is

\[
\begin{align*}
G_A^s &= -\alpha_A I_A + (K_A(I_A) - \beta_A I_A - (1 - \alpha_A) I_A) \tau_B + \beta_A I_A \pi_A e^{-\tau} \\
G_B^s &= -\alpha_B I_B + (K_B(I_A, I_B) - \beta_B I_B - (1 - \alpha_B) I_B) \tau_B + \beta_B I_B \pi_B e^{-\tau}
\end{align*}
\]  

(21)

If firm B is not eligible for subsidies (i.e., \( \alpha_B = \beta_B = 0 \)), then the government’s net claim on B is

\[
G_B(I_A, I_B, 0) = (K_B(I_A, I_B) - I_B) \tau_B e^{-\tau}
\]  

(22)

From the standpoint of social welfare, the government is justified in supporting project A if project A is rejected solely on the basis of tax considerations, and if the combined net receipts from A and B with subsidies are positive, \( G_A^s + G_B^s > 0 \).

We can define the combined net present values of firms A and B from the new investments for the no-tax case as the “value added to the economy” that results from the investments, \( H = NPV_A + NPV_B \). The value added to the economy if taxes (and subsidies) are imposed can be defined as

\[
H^S = NPV_A^S + NPV_B^S + G_A^S + G_B^S
\]  

(23)

The added value of \( H^S \) measures the present value of the total net rent that accrues to the economy as a result of accepting the two investments.

Further, we can show that if firm A cannot charge firm B for its positive impact on B’s cash flow, and if this spillover effect is substantial, then the government may be justified in encouraging A to invest more than what would otherwise be optimal, \( I_A^* \). The government, rather than Firm A, can capture part of the spillover effect through taxes, and a social welfare outcome can be achieved.

To illustrate these claims, we continue the example from Section III, and assume that firm A has a production function described in Equation (17), with parameters specified above. We assume that Firm B has a production function similar to Equation (17), but we add the assumption that the relevant investment is \( I_B \) and a fraction \( \delta \) of \( I_A \). Thus,
\[ K_B(I_A, I_B) = D_1 - D_2 e^{-\frac{I_A + \delta B}{D_3}} \] (24)

\( \delta \) represents the spillover factor, and \( D_1, D_2, \) and \( D_3 \) are known parameters.

If we assume that for firm B, \( D_1=600, D_2=1800, \) and \( D_3=100, \) and for firm A, that \( C_1=600, C_2=1200, \) and \( C_3=100, \) as described in section III, then we can show that assuming \( \pi_A=0.6, \) and \( \pi_B=0.8, \) and the spillover factor \( \delta=0.1, \) without taxes and subsidies the combined NPV of firms A and B is 158.1. With a 40% tax rate, firm A rejects the investment and firm B does not benefit from A’s aborted investment. In this case the NPV of B is only 29.8, while that of A is zero, and the government’s receipts, in present value terms, come to 60.3. Hence, the value added to the economy (i.e., \( H^S = NPV_A^S + NPV_B^S + G_A^s + G_B^s \)) is only 90.1.

With a subsidy rate \( \alpha_A \) of 10%, firm A invests 176.5 (compared to 191.4 for the no tax case), and adds 10.8 to its NPV. Firms A and B together attain an NPV of 53.0, and \( G^S=101.2. \) The total added value is 154.2. With a subsidy rate of 20%, the total added value is 156.5, which approximates to the combined NPV for the no-tax case. Obviously, the allocation of values among A, B, and the government changes with each tax/subsidy combination.

Typically, the Government’s economic objective is to collect a fixed amount \( G^S = \bar{G} \) (or, \( G_A^S + G_B^S = \bar{G} \)), where \( \bar{G} \) is the assumed, fixed government budget and maintain an economically optimal level of investment, i.e., \( I_A=I_A^*, \) and \( I_B=I_B^*, \) where asterisks denote the optimal level of investment for the no-tax case. The assumption is that the optimal level of investment \( I^* \) yields optimal output from a social welfare perspective.

We note that with a spillover effect that cannot be internalized, firm A invests less than the socially optimal level, since it ignores the effect on the NPV of firm B. In such a case, subsidizing A may cause it to invest more than \( I_A^* \), and thus capture, in social welfare terms, the impact of A on NPV_B.

We assume that government must collect \( \bar{G} \) to finance its activities (and \( \bar{G} \) is determined exogenously to the model). The government can affect \( I_A, I_B, \) and \( G_A^S + G_B^S \) by changing both the tax rate, \( \tau, \) and the subsidy rate, \( \alpha. \) We assume that the tax rate is uniform for all firms and is more difficult to change than \( \alpha. \) In most cases, the subsidy rate can be altered with greater flexibility, and differentially across industries.

In Figure 3 we show the total value added (i.e., \( H^S = NPV_A^S + NPV_B^S + G_A^S + G_B^S \)) and government receipts \( G_A^S + G_B^S \) as a function of \( \alpha_A, \) the subsidy rate for firm A. At low levels of \( \alpha, \) firm A still rejects the investment. Only if \( \alpha>1.9% \) does firm A accept the investment. As a result, both total added value, \( H^S, \) and government receipts (in present value terms), \( G^S, \) jump substantially, with the increase in \( H^S \) being higher than the increase in \( G^S. \)
In our numerical example, if the government has a fixed target of $G = 100$, it can provide a subsidy rate of $\alpha = 11.0\%$. In this case the resulting investment and values are presented in Table 1.

Figure 4 depicts isocurves of $G_S$ and $H_S$ on the $\alpha_A, \alpha_B$ plane. The objective is to check the trade-off between subsidizing A and B and the optimal allocation of the subsidy budget that can maximize the social welfare function, $H_S$, subject to budgetary constraint, $G$. The isocurve $G_S=100$ depicts all the pairs $\alpha_A, \alpha_B$ such that the total government claim is 100. In a similar way the isocurve $H_S=154.5$ depicts all pairs of $\alpha_A, \alpha_B$ (given optional investment decisions by equity holders) such that the total added value, $H_S$, is equal to 154.5.

In Figure 4 we look for the optimal subsidy policy ($\alpha_A, \alpha_B$) for a given $G$, that maximizes $H_S$. The figure shows that given our assumptions, the optimal policy is usually to support A and not B. This finding results from the assumption that there is a spillover effect from A to B.

For example, for $G = 105$, the highest $H_S$ that can be achieved is 153.2, and then $\alpha_A=6.6\%$ and $\alpha_B=0$. For $G = 100$, the highest total added value is $H_S=154.5$.

To achieve this level, the government should subsidize A with $\alpha_A=11\%$. The optimal subsidy to B is almost nil. For $G = 95$, $\alpha_A=15.1\%$ and $\alpha_B$ equals zero, the highest total added value is $H_S=155.5$.

V. The Decision to Subsidize High-Tech versus Low-Tech Projects

It is a common view that if a government decides to subsidize investments, it should subsidize high-tech, rather than low-tech, companies. The usual explanation is that high-tech companies face higher risk. In addition to the commercial risks encountered by all firms, they face a technological risk and the uncertainty of introducing new products on the market. Without subsidies, so the claim goes, high-tech companies will reject risky projects, opting instead for less risky alternatives. The rationale for subsidizing high-tech ventures is also supported by the claim that high-tech firms generate greater externalities than do low-tech firms.6

We introduce the differences between high-tech (HT) and low-tech (LT) firms to the model of the firm in Equation (1). The first distinction is reflected in the slope of the revenue function, $K(I)$, which we assume is steeper for HT than for

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6 This viewpoint is strongly emphasized by the Department of Finance and Revenue, Canada in their report (December 1997).
For any given investment, \( I \), we assume that if successful, HT yields greater revenues.

The probability that HT will succeed is smaller than the probability for LT. We note that the certainty equivalent of the investment grant, \( \alpha \), and royalty rate, \( \beta \), is greater for a lower probability of success, since \( \alpha \) is received in advance as a function of the investment, but the obligation to pay \( \beta \) is contingent on future performance. The contingent royalty is ex-ante uncertain, and the higher the uncertainty, the smaller its present value.

The third difference is in the spillover effect, which we expect to be relatively small for the LT firm. The government acts as an economic agent that is required to enhance welfare under a given corporate tax code, and with a limited, fixed budget to subsidize investment activity. In our partial equilibrium framework, we focus on the decision of the government to subsidize either LT or HT, when the two firms are differentiated by the slope of \( K(I) \), the discount factor \( \pi \), and the spillover factor, \( \delta \).

In Figure 5 we describe the expected cashflow function, \( K(I) \), for HT and LT firms.\(^7\) For continuity, we assume that HT is firm A and LT is firm B from the previous section. We also assume that the probability for LT is 0.8 compared to 0.6 for the HT firm. We assume that the spillover factor for HT, \( \delta \), is 0.1, i.e., each dollar invested in HT generates an additional 0.1 dollar at B. We also assume that no reverse spillovers from LT to HT exist.

Table 2 summarizes the results for HT and LT under the above assumptions. With no taxes or subsidies, the two firms produce a risk-adjusted total NPV of 158.1 (see column 9). We note that 158.1 is the socially optimal level if the two firms cannot collude.

Figure 6 shows that a higher level of NPV can be achieved if the spillover effect is internalized and firm A invests more than 191.4. For example, inducing firm A to invest 201.9 by providing it with a subsidy of \( \alpha=0.1 \), will increase the total NPV of the two firms from 158.1 to 178.8 by more than the subsidy paid to A by the government (20.2), so that H can be set at 158.6.

\(^7\) A detailed numerical example of the effect of taxes and subsidies will be provided by the authors on request.
for a total net added value, \( H^S \), of 90.1. The government can restore the social welfare close to its maximum value by subsidizing HT investments by 30%, as shown in the last row of Table 2.

Once again, it is better to subsidize HT than LT, given the assumptions outlined above and a budget constraint on \( G \). This preference for high tech is based on the higher risk associated with high-tech ventures, the greater sensitivity to tax rates, and on the outward direction of economic spillover. With a tax rate of 40% (and no subsidies), HT rejects projects and LT continues to invest. Hence, the probability of underinvestment in high-tech industries is higher than in low-tech industries. Given the direction of economic spillovers, the economy risks losing more from the standpoint of cumulative social welfare if subsidies are withheld from high-tech enterprises.

**VI. Summary and Conclusions**

Our paper focuses on governments’ economic decisions to subsidize investments in the private sector, and to discriminate among firms in their support programs. Our assumption is that by taxing corporate profits, the government affects the investment decisions of firms and causes entrepreneurs to invest less than an amount that is socially optimal. Therefore, investments desirable from a social standpoint may be rejected by shareholders. Withholding investment may also lead to the collection of fewer taxes.

Our analysis and conclusions are based on a new approach to measuring social welfare in an uncertain environment. We calculate the present value of cash flows to the entrepreneur plus the present value of net government receipts, discounted at rates that account for the investment risk. Consistent with the analysis of social welfare gains, we model externalities and measure their uncertain impact in terms of present value.

Given corporate taxation, one possible economic solution for enhancing public welfare and simultaneously increasing government tax revenues is to subsidize investments. The primary advantage of investment subsidies over tax cuts stems from subsidies’ flexibility, which gives governments the ability to discriminate among industries, firms, and type of activity.

We analyze a scheme under which the government provides investment grants and charges royalties from future revenues, and show the conditions under which it is rational to subsidize investments initiated by the private sector. Such a scheme can reduce problems of asymmetric information between the owners of the firm and the government. We show that by calibrating the sizes of the grants (\( \alpha \))
and the royalty rates ($\beta$), the government can partially restore socially optimal investment levels.

In the second part of the paper we look at the government’s preference for supporting investments in high-tech, rather than low-tech, firms. We introduce spillover effects (or externalities) to the model and analyze these effects. Our basic assumption is that high-tech firms have more significant externalities than do low-tech firms. Thus, high-tech firms tend to suffer from problems of underinvestment to a greater degree.

We show how the government should allocate subsidies between the two types of firms to enhance total social welfare, when the government is constrained by the need to achieve fixed revenues. Under reasonable assumptions, the lion’s share of support should be given to the high-tech firm. With the spillover effect, which cannot be captured by the originator, subsidies can augment the benefits of better investment decisions from the standpoint of social welfare.
References


**Table 1**: Optimal investments, NPVs, Government Net Value ($G^S$), and Total Added Value ($H^S$) for the 2-firms case with spillover, when subsidy rate is $\alpha=11\%$.

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>177.6</td>
<td>227.7</td>
<td>405.3</td>
</tr>
<tr>
<td>NPV$^S$</td>
<td>12.2</td>
<td>42.3</td>
<td>54.5</td>
</tr>
<tr>
<td>$G^S$</td>
<td>34.4</td>
<td>65.6</td>
<td>100.0</td>
</tr>
<tr>
<td>$H^S$</td>
<td></td>
<td></td>
<td>154.5</td>
</tr>
</tbody>
</table>

The table shows the optimal investments, NPVs, government net receipts in present value terms and total added value ($H^S = G^S + \text{NPV}^S$) for the two-firms case, with a spillover factor $\delta=10\%$, when the government has a fixed target of $G^S=100$. In order to achieve the target, with tax rate $\tau=30\%$ the government should pay a subsidy rate of $\alpha=11\%$ to firm A. For this case the combined NPV of A and B is 54.5 and the total added value to the economy from the two new investments is 154.5.
Table 2: Optimal investment (I), NPV of the firm (NPV), government net benefit (G), and total added value (H) from the High-Tech (HT) and Low-Tech (LT) companies, for different tax rates (τ) and subsidies (α).

<table>
<thead>
<tr>
<th></th>
<th>HT</th>
<th></th>
<th></th>
<th>LT</th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>NPV</td>
<td>G</td>
<td>I</td>
<td>NPV</td>
<td>G</td>
<td>NPV</td>
<td>G</td>
</tr>
<tr>
<td>τ=0, α_{HT}=0</td>
<td>191.4</td>
<td>47.6</td>
<td>0</td>
<td>241.6</td>
<td>110.5</td>
<td>0</td>
<td>158.1</td>
<td>0</td>
</tr>
<tr>
<td>τ=0, α_{HT}=10%</td>
<td>201.9</td>
<td>67.3</td>
<td>-20.2</td>
<td>240.5</td>
<td>111.5</td>
<td>0</td>
<td>178.8</td>
<td>-20.2</td>
</tr>
<tr>
<td>τ=30%, α_{HT}=0</td>
<td>174.3</td>
<td>9.5</td>
<td>36.6</td>
<td>233.2</td>
<td>58.5</td>
<td>49.7</td>
<td>68.0</td>
<td>86.3</td>
</tr>
<tr>
<td>τ=30%, α_{HT}=10%</td>
<td>184.9</td>
<td>24.4</td>
<td>23.0</td>
<td>232.2</td>
<td>59.3</td>
<td>50.0</td>
<td>83.7</td>
<td>73.0</td>
</tr>
<tr>
<td>τ=40%, α_{HT}=0%</td>
<td>0</td>
<td>0(-2.4)</td>
<td>0</td>
<td>245.5</td>
<td>29.8</td>
<td>60.3</td>
<td>29.8</td>
<td>60.3</td>
</tr>
<tr>
<td>τ=40%, α_{HT}=10%</td>
<td>176.5</td>
<td>10.8</td>
<td>35.6</td>
<td>227.9</td>
<td>42.2</td>
<td>65.6</td>
<td>53.0</td>
<td>101.2</td>
</tr>
<tr>
<td>τ=40%, α_{HT}=30%</td>
<td>201.6</td>
<td>40.0</td>
<td>7.1</td>
<td>225.3</td>
<td>43.9</td>
<td>66.3</td>
<td>83.9</td>
<td>73.4</td>
</tr>
</tbody>
</table>

The table shows the optimal investments and their NPVs and the government’s expected net receipts in present value terms for alternative tax and subsidy rates for the two-firms case. For example, the last line of the table shows the results for tax rate of 40% and subsidy rate of 30%, where the subsidy applies to the high-tech firm only. For the high-tech firm the optimal investment is 201.6 and it generates NPV of 40, while the government has net receipts from this high-tech firm equal 7.1. The second (low-tech) firm, should invest 225.3, yielding an NPV of 43.9, and the government’s benefit is 66.3. In total, the firms produce NPV (after tax and subsidy) of 83.9, and the government’s total net benefit is 73.4, yielding a total social welfare benefit to the economy of 157.4 from accepting the two projects. For tax rate of 40%, with no subsidies, it is shown in the table that the total added value is only 90.1.
Figure 1. The marginal cash flow $K'(I)$ as a Function of the investment I. For the no tax Case the optimal investment is $I^*$ and the Marginal cashflow is $\frac{e^r}{\pi}$ . For the tax case the optimal investment is $I^{**}$ and the marginal cashflow is $\frac{e^r 1 - \tau \pi e^{-r}}{\pi (1 - \tau)}$ .
Figure 2. The NPV as a function of investment size $I$ for 3 cases: no tax, taxes but no subsidies, taxes and subsidies. The tax (and no subsidies) case is based on equation (1). The no-tax case is also based on equation (1) when $\tau=0$. The case of taxes and subsidies is based on Equation (6). The parameters on which the graph is based are: $K(I)=600-1200\cdot\exp(I/100)$, $\pi=0.6$, $r=6\%$, $\tau=40\%$, $\alpha=0.3$, and $\beta=0.2$. 
Figure 3. The total added value, \( H \), and government value, \( G \), as a function of the subsidy rate, \( \alpha_A \) (when \( \beta_A = \beta_B = \alpha_B = 0 \)) for the two-projects case with a spillover effect (\( \delta = 0.1 \)). \( \bar{G} \) describes the assumed minimal required government budget (100). Functions \( H \) and \( G \) are based on Equations (21) and (23). Below \( \alpha_A = 1.9\% \), project A is rejected, hence the total value added and government present value are low. For \( 1.9\% < \alpha_A < 11\% \), project A is accepted. This has an impact on the optimal investment in B, and government value \( G \) is still above \( \bar{G} \). For \( \alpha_A > 11\% \) there is oversubsidation and \( G \) falls below \( \bar{G} \).
Figure 4. Isocurves of $G$ and $H$ on the $\alpha_A$, $\alpha_B$ plane. The figure describes the trade-offs between subsidizing project A, by $\alpha_A$, and B by $\alpha_B$. The effects of subsidies on both the total value added, $H$ (Equation (23)) and on government value $G$ (equation (21)) are depicted by the isocurves. For example, the curve $H=154.5$ depicts all pairs of $\alpha_A$, and $\alpha_B$ that bring the total added value function to be equal 154.5. The curve $G=100$ depicts all pairs of $\alpha_A$ and $\alpha_B$ for which $G=100$. The tangency between the curves $H=154.5$ and $G=100$ corresponds to the optimal point ($\alpha_A$, $\alpha_B$) for which $G=100$ and the total value added is maximized.
**Figure 5.** Comparison of profit functions $K$ for the High-Tech firm (HT) and for the Low Tech firm (LT). The functions are given by equations (9) and (9B) with $c_1=600$, $c_2=1200$, and $c_3=100$, $\pi_{HT}=0.6$ for HT and with $c_1=600$, $c_2=1800$, and $c_3=100$, $\pi_{LT}=0.6$ for LT.
Figure 6. Total added value, $H$ for HT and LT companies as a function of the tax rate $\tau$, and the subsidy rate $\alpha_{HT}$. The figure shows the combined effects and trade-offs between the corporate tax rate and the subsidy for HT, on the total added value. For high tax rates both investments are rejected and $H=0$. At $\tau=0$, there still may be an incentive for subsidizing HT due to its spillover effect, since $H$, will go up with $\alpha_{HT}$ over a certain range.