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Modeling FX risk adjusted duration of foreign government bonds taking the viewpoint of a domestic investor interested in converting cash flows into domestic currency, we show that foreign bond portfolio managers who fail to adjust for FX risk in calculating duration of foreign sovereign debt observe duration with error and consequently engage in inefficient active rate-anticipation and immunization strategies. Using daily bond returns for sovereign issuers in five currencies observed between January 2000 and June 2005, we estimate FX-adjusted elasticity for different maturities. Our empirical findings indicate that FX-adjusted elasticity is significantly different from Macaulay unadjusted duration for all countries and all maturities. [G15]

■Although duration is an important tool for the risk management of fixed income securities, research has not explored how it may be applied in the important arena of foreign bond investment. In the present study, we address this gap in the literature by examining the foreign exchange (FX) risk adjustment required for duration, and its impact on the efficiency of active rate-anticipation and passive immunization strategies for portfolios of foreign bonds. Our paper builds on Dym's (1991, 1992) demonstration that the use of Macaulay duration is inefficient for international bond portfolios because duration assumes that the bonds issued by different governments are priced in similar interest rate environments. When FX risk is unhedged, he shows that the riskiness of a foreign bond consists of global and local interest rate risk, the country's exchange rate risk, and their interaction. This finding is consistent with a study by Chow, Lee, and Solt (1997) who examine the FX risk exposure of US stocks and bonds. They find that the impact of exchange rate changes on bond returns is predominantly driven by the correlation between exchange rate and interest rate changes.

Where Dym (1991, 1992) rejects duration in favor of an alternative risk measure, in this paper, we derive a model for the FX risk adjusted duration of foreign government bonds taking the viewpoint of a domestic investor holding an unhedged position in foreign sovereign bonds and interested in converting cash flows offered by the foreign bond into domestic currency. Such a model is useful as it builds on the widespread acceptance of duration by academics and practitioners. We show that *ex ante* FX risk adjusted duration is the sum of a bond's Fisher-Weil duration and the elasticity of a hypothetical hedging portfolio of forward FX contracts with respect to shifts in the term structure of the foreign issuer. We further demonstrate that two conditions must hold for the difference between the two duration measures to be trivial making appropriate the use of ordinary (unadjusted) duration. First, the foreign country must react to shifts in the domestic country's term structure and conduct a 1:1 adjustment of its interest rate. Second, the spot exchange rate must be inelastic with respect to shifts in the domestic interest rate. This second condition is in agreement with the work of Dym (1991, 1992) and Chow, Lee, and Solt (1997).

Given that our two conditions are quite strong, the implication of our model is that foreign bond portfolio managers who fail to adjust for FX risk in calculating duration of foreign sovereign debt observe duration with error and consequently engage in inefficient active rate-anticipation strategies. Such errors also impact

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passive immunization strategies which may suffer substantial losses.

In order to test the implications of our model empirically, we adopt the perspective of a US based investor who invests in bonds issued by various foreign governments. Using a regression methodology developed by Fons (1990) and daily bond returns for sovereign issuers in five currencies observed between January 2000 and June 2005, we estimate the FX-adjusted elasticity for several countries and different maturities.[1] In agreement with our theoretical results, our empirical findings indicate that FX-adjusted elasticity is significantly different from Macaulay unadjusted duration for all countries and all maturities.

We then proceed to decompose FX-adjusted elasticity empirically into two components as suggested by our theory: the elasticity of the foreign bond denominated in the foreign currency and the elasticity of the FX rate with respect to the US dollar. The reference for both elasticities is the US Treasury yield for three benchmark maturities. This decomposition shows that, in general, for most countries the elasticity of the FX rate dominates. This is in agreement with the findings of Chow, Lee, and Solt (1997) for an earlier period (1977-1989) that the impact of changes in exchange rates on bond returns is driven mainly by the correlation between shifts in exchange rates and interest rates. The main conclusion of our empirical test is that the FX-adjustment required for duration of foreign bonds is both statistically significant and economically nontrivial for a US investor. Ignoring this adjustment, and using Macaulay duration instead, may lead to substantial losses.

The paper is organized as follows: In Section I, we present our model for foreign exchange risk adjusted duration, followed by the implications of our model in Section II. In section III, we test the empirical implications of our model. Section IV concludes.

I. Model for the Foreign-Exchange Risk Adjusted Duration

A. The Bond Pricing Equation

Consider a T-years-to-maturity, coupon-bearing, foreign government bond, offering default-free annual payments $C_{f,t}$, $t = 1,2,\ldots,T$. The foreign bond's cash flows, denoted by subscript f, are denominated in the home currency of the issuing foreign government. Assume that the bond is option-free with no sovereign risk. The present value of this bond in the foreign currency for a bondholder in its home country, V_f, is given by:

$$V_f = \sum_{t=1}^{T} C_{f,t} \exp\{-t r_{f,t}\} \tag{1}$$

where $r_{f,t}$ is the continuously compounded yield on a t-year zero-coupon foreign-government bond.

A domestic investor interested in converting cash flows offered by the foreign bond into the domestic currency can eliminate the bond's foreign exchange risk by creating a *covered position*. That is, the domestic bondholder will hold the foreign bond along with a series of foreign exchange forward contracts corresponding to the payments made by the foreign bond, each removing foreign exchange risk associated with converting a coupon payment into the domestic currency. Let ϕ_t denote the t-period forward domestic currency/foreign currency exchange rate, then the value of the covered position in the foreign bond for the domestic bondholder in the domestic currency, V_d, is given by:

$$V_d = \sum_{t=1}^{T} C_{f,t}\phi_t \exp\{-t r_{d,t}\} \tag{2}$$

where $r_{d,t}$ is the continuously compounded yield on a t-year, zero-coupon, domestic government bond.

B. The Foreign Exchange Risk Adjusted Duration

To measure the price elasticity of the foreign bond with respect to shifts in the domestic riskless term structure of interest rates, we derive a foreign exchange risk adjusted duration for foreign government bonds. Given bond pricing equation (2), we employ the standard price elasticity definition of duration when returns are continuously compounded: $D_d = -\frac{1}{V_d}\sum_{t=1}^{T}\left(\frac{\partial V_d}{\partial r_{d,t}}\right)$, and after rearranging terms, we obtain our foreign exchange risk adjusted duration, from the domestic investor's point of view:

$$D_d = \frac{\sum_{t=1}^{T}\left(t - \frac{1}{\phi_t}\frac{\partial \phi_t}{\partial r_{d,t}}\right) C_{f,t}\phi_t \exp\{-t r_{d,t}\}}{V_d} \tag{3}$$

According to the law of one price, the current value of a domestic bondholder's covered, domestic currency position in the foreign bond must be equivalent to the value of this bond for a bondholder in the home country, converted at the spot exchange rate: $V_d = V_f\phi_0$.[2] Invoking interest rate parity, the forward foreign

[1] Fons uses this methodology to empirically estimate the effective duration of callable corporate bonds.

[2] Our analysis uses nominal exchange rates as we focus our modelling on the portfolio manager's decision. Chow, Lee and Scott (1997) employ a weighted index of real exchange rates in their study of how exchange-rate shifts impact real bond and stock returns.

exchange rate is given by: $\phi_t = \phi_0 \exp\{t(r_{d,t} - r_{f,t})\}$. Thus, we can rewrite equation (3) as:

$$D_d = \frac{\sum_{t=1}^{T}\left(t - \frac{1}{\phi_t}\frac{\partial \phi_t}{\partial r_{d,t}}\right) C_{f,t} \exp\{-tr_{f,t}\}}{V_f} \quad (4)$$

Let D_f denote the price elasticity for the foreign bond with respect to the home country's riskless term structure of interest rates (i.e., D_f is the Fisher-Weil duration), then

$$D_f = -\frac{1}{V_f}\sum_{t=1}^{T}\left(\frac{\partial V_f}{\partial r_{f,t}}\right) = \sum_{t=1}^{T} t \frac{C_{f,t} \exp\{-tr_{f,t}\}}{V_f}.[3]$$

Also, let D_{ϕ_t} denote the elasticity of the t-period forward domestic currency/foreign currency exchange rate with respect to the domestic riskless term structure, then $D_{\phi_t} = -\frac{1}{\phi_t}\frac{\partial \phi_t}{\partial r_{d,t}}$. Substituting these elasticity definitions into equation (4), we get

$$D_d = D_f + D_\phi \quad (5)$$

where

$D_\phi = \sum_{t=1}^{T} D_{\phi_t} \frac{C_{f,t} \exp\{-tr_{f,t}\}}{V_f}$, the value-weighted duration of the portfolio of forward contracts in the covered position.

From equation (5) it is clear that the foreign exchange (FX) risk adjusted duration given in equation (3), D_d, is different from the duration calculated from the point of view of an investor based at the home country of the issuing foreign government, D_f. The former measures the price elasticity of the foreign bond with respect to shifts in the domestic term structure of interest rates, while the latter measures price elasticity of the foreign bond with respect to shifts in the term structure of interest rates of the foreign issuer. The difference between the two duration measures is given by the elasticity of a portfolio of foreign exchange forward contracts corresponding to the payments made by the foreign bond, D_{ϕ_t}, each removing foreign-exchange risk associated with converting a coupon payment into the domestic currency.

From a practical point of view, a foreign bond portfolio manager using the unadjusted duration, D_f, to measure the bond's elasticity with respect to shifts in the domestic term structure of interest rates can do so only as long as the elasticity of the portfolio of foreign-exchange forward contracts is trivial. Otherwise, measures such as the Fisher-Weil duration of the foreign bond will fail to correctly predict changes in the portfolio value induced by shifts in the local term structure.

[3]Fisher-Weil duration extends Macaulay duration by introducing a term structure (Bierwag, 1987a, 1987b).

II. Implications of Our FX Risk-Adjusted Duration Model

Next, we examine the nature of the relationship between our FX risk adjusted duration (with the domestic riskless yield as the reference rate) and the ordinary Macaulay duration of the foreign bond (with respect to its own foreign yield). In particular, we show that interest rate policy in the foreign country and its relation to domestic policy will determine the nature of this relationship. The sensitivity of the spot FX rate to shifts in the domestic term structure is also a determinant of this relationship. Thus, ignoring these determinants may result in an inefficient immunization strategy. Foreign bond portfolio managers who fail to adjust for FX risk and utilize active rate-anticipation duration strategies may suffer substantial losses.

A. The FX Risk-Adjusted Duration vs. Macaulay Duration – A Theoretical Examination

In order to make the analysis more manageable, we modify our assumptions from the previous section to reflect flat term structures in both countries.[4] Thus, the FX risk adjusted duration is given by $D_d = -\frac{1}{V_d}\frac{\partial V_d}{\partial r_d}$. This is a special case of our duration measure given in equation (4). Substituting $V_d = V_f\, \phi_0$ on the right-hand-side of this FX-adjusted elasticty, produces the following relationship:

$$D_d = \frac{\partial r_f}{\partial r_d} D_f + D_{\phi_0} \quad (6)$$

where

$D_f = -\frac{1}{V_f}\frac{\partial V_f}{\partial r_f}$ = the bond's ordinary Macaulay duration with the foreign yield as the reference yield,

r_f = the foreign bond's flat yield to maturity, and

D_{ϕ_0} = the elasticity of the spot domestic-currency/foreign-currency exchange rate with respect to the domestic riskless term structure,

$$D_{\phi_0} = -\frac{1}{\phi_0}\frac{\partial \phi_0}{\partial r_d}.$$

Thus, equation (6) implies that differences between the two duration measures will be trivial, and one can use the ordinary Macaulay duration only when the following two conditions hold: (1) the foreign country reacts to shifts in the domestic country's term structure

[4]This assumption reduces our general formulation to a single-factor model. We examine its explanatory ability below.

and conducts a 1:1 adjustment of its interest rate, and (2) the spot rate is inelastic with respect to shifts in the domestic interest rate.[5] This means that the FX adjustment for duration will be smaller the greater the extent to which a foreign government reacts to interest rate changes in the investor's home country. For example, for U.S.-based investors, the adjustment will be relatively smaller for Canada than for the European Community. We test this implication below.

B. The Importance of the FX Risk Adjustment

In this section, we discuss the potential financial damage of failing to adjust duration for FX risk for active and passive investors in turn. Assessing such damage is important in establishing the relevance of our adjustment in practice. Starting with active managers, and using a first-order Taylor series expansion, we show that the relationship between a percentage change in the foreign bond price denominated in the domestic currency $\left(\frac{\Delta V_d}{V_d}\right)$ and changes in the domestic continuously compounded riskless interest rate (Δr_d) is well approximated by:[6]

$$\frac{\Delta V_d}{V_d} = -D_d\,\Delta r_d \tag{7}$$

Assuming our model is correct and an investor is incorrectly using unadjusted duration, D_f, rather than an FX risk-adjusted duration, D_d, to measure the sensitivity of the domestic value of the bond to changes in the domestic riskless term structure, then one can express equation (7) as

$$\frac{\Delta V_d}{V_d} = -D_f\,\Delta r_d - error\,\Delta r_d$$

Where *error* represents the difference between the FX risk adjusted duration and its Macaulay counterpart ($D_d - D_f$). For a 1% change in the domestic riskless yield, our investor will anticipate a percentage change in the domestic value of the foreign bond which differs by $D_d - D_f$ percent from the change approximated by equation (7). From equation (6) we know that: $D_d - D_f = \frac{\partial[r_f - r_d]}{\partial r_d} D_f + D_{\phi_0}$. Thus, the magnitude of the error of ignoring the FX adjustment is determined by the relation between the interest rate policies in the two countries, as well as by the exchange rate sensitivity to shifts in the domestic term structure.

[5]Equation (6) can be rederived for an uncovered position and the details are available upon request from the authors.

[6]A note on risk-adjusted convexity is available upon request from the authors.

In addition to its use in active rate-anticipation strategies, duration is widely employed in hedging or immunizing portfolio returns over a given planning horizon or for a given liability structure. To demonstrate the immunization implication of our model, we take the viewpoint of an investor who wishes to form a portfolio of foreign bonds with a horizon of k years (because in k years the investor needs to fulfill a debt obligation).[7] To achieve immunization the manager needs to set $k = D_d = D_f + D_{\phi_t}$ or, in words select a portfolio with a duration equal to that of the planning horizon *taking the foreign exchange elasticity adjustment into account.* It follows that investors in foreign bonds who ignore the impact of currency risk on the value of foreign bonds (and therefore set their horizon to D_f rather than D_d) will not be holding immunized portfolios.

III. Is the FX Risk Adjustment Important?

A. Data and Methodology

In order to test the empirical implications of our FX risk adjustment for duration, we adopt the perspective of a US based investor in bonds issued by foreign governments. Since in our model, bonds bear no sovereign risk, we conduct tests on high quality sovereign issues with five currencies: Australian, Canadian, Danish, British, and the Euro. We use daily strip rates from Bloomberg, derived from benchmark curves, for all countries, including the U.S., for maturities of 2, 5, and 10 years, covering the period between January 1, 2000 and June 28, 2005.[8] Daily foreign exchange rates are also obtained from Bloomberg.

Our first test is designed to estimate the FX-adjusted duration for each country for the above maturities. This will allow us to compare the adjusted duration to the unadjusted (Macaulay) duration, and calculate the error of ignoring this adjustment. To this end we run the following regression model:

$$\ln(V_{d,T,t}) = a_1 + b_1\, r_{d,T,t} + e_{1t},\quad t = 1,\ldots,t_n,\; T = 2, 5, 10, \tag{8}$$

where

$V_{d,T,t}$ = the value of a T-year foreign government strip on day t, converted to US dollars:

[7]A mathematical proof in the spirit of Bierwag and Khang (1982) is available upon request.

[8]The Euro strip index is from the benchmark curve calculated from bonds issued by Euro governments. Markets in different countries are closed due to holidays on different dates. Our sample for each country includes only daily observations for days where both the US market and the market in the issuing country are open. As a result, our sample periods range from 1407 days for the Euro countries to 1426 for Canada.

$V_{d,T,t} = \phi_{0,t} V_{f,T,t} = \phi_{0,t} \exp\{-Tr_{f,T,t}\}$,

$V_{f,T,t}$ = the value of a T-year foreign government strip on day t, denominated in the currency of the issuing government

$\phi_{0,t}$ = the spot FX rate on day t

$r_{f,T,t}$ = the continuously compounded yield on a T-year foreign government strip index at time t (calculated as ln[1 + observed yield on index]

$r_{d,T,t}$ = the continuously compounded yield on a T-year US Treasury strip index at time t (calculated as ln[1 + observed yield on index]

a_1 = intercept

b_1 = slope

e_{1t} = error term

n = number of daily observations

In regression model (8), the absolute value of the slope estimator, $-\hat{b}_1$, is the empirical FX-adjusted elasticity of the foreign bond.[9] The FX-adjusted duration estimator allows us to test whether this measure, $D_{d,T}$, is significantly different from Macaulay duration, $D_{f,T}$. Since we use data on strips, it is always the case that: $D_{f,T} = T$. Thus, the test is whether $-\hat{b}_1$ is significantly different from T for a T-year strip index.

Regression model (8) tells us whether duration, adjusted for FX risk from the point of view of a US investor, is different from Macaulay duration. It also provides an estimate of the magnitude of the error caused by ignoring the FX adjustment ($D_d - D_f$). Recall that equation (6) implies that the FX adjustment applied to Macaulay duration has two elements: 1) the elasticity of the foreign bond, valued in the foreign country's currency, with respect to the US interest rate. and 2) the elasticity of the spot FX rate with respect to the domestic interest rate. We represent equation (6) as:

$$D_d = \left(-\frac{1}{V_f}\frac{\partial V_f}{\partial r_d}\right) + \left(-\frac{1}{\phi_0}\frac{\partial \phi_0}{\partial r_d}\right). \quad (9)$$

Equation (9) offers a convenient decomposition of the FX-adjusted duration facilitating analysis of the relative importance of each of the two elements. We estimate these two components of the FX-adjusted duration with the following two regression models:

$$\ln(V_{f,T,t}) - a_2 + b_2 r_{d,T,t} + e_{2t},\quad t = 1,\ldots,t_n,\ T = 2, 5, 10, \text{ and} \quad (10)$$

$$\ln(\phi_{0,t}) - a_3 + b_3 r_{d,T,t} + e_{3t},\quad t = 1,\ldots,t_n,\ T = 2, 5, 10, \quad (11)$$

where a_2 and a_3 are intercepts, b_2 and b_3 are slope coefficients, and e_{2t} and e_{3t} are regression error terms. Paralleling regression model (8), the absolute value of the slope estimators in regression models (10) and (11), $-\hat{b}_2$ and $-\hat{b}_3$, are the estimates of the two components of the FX-adjusted elasticity, as in equation (9). This is because:

$$b_2 = \frac{\partial \ln(V_{f,T,t})}{\partial r_{d,T,t}} = \frac{1}{V_{f,T,t}}\frac{\partial V_{f,T,t}}{\partial r_{d,T,t}} \text{ and}$$

$$b_3 = \frac{\partial \ln(\phi_{0,t})}{\partial r_{d,T,t}} = \frac{1}{\phi_{0,t}}\frac{\partial \phi_{0,t}}{\partial r_{d,T,t}}.$$

Note that by definition and in agreement with equation (9), the regressions are constrained such that: $\hat{b}_1 = \hat{b}_2 + \hat{b}_3$. Regression models (10) and (11) allow us to estimate the two components of the FX-adjusted duration and assess their relative importance.

B. Empirical Results

In Exhibit 1, we report the estimates for regression model (8).[10] For every country, the exhibit shows the estimates of the regression parameters with their t-values, the adjusted R^2, the standard error of the slope estimator, and the t-statistic for the null hypothesis that the absolute value of the slope is different from time to maturity, T, which is the Macaulay duration of the foreign strip. If the null hypothesis is rejected, we may conclude that the magnitude of the error caused by ignoring the FX adjustment $(D_d - D_f) = (D_d - T)$ is significant for the examined country. For each country the results are reported for maturities of 2, 5, and 10 years.

The exhibit reports that the estimated FX-adjusted elasticity (with respect to the US Treasury rate) of all maturities in all countries is greater than Macaulay (unadjusted) duration. In all cases this result is statistically significant at the one percent level (except for the 10-year strip in Great Britain where the one-tail test is significant at the 6% level). Furthermore, the error, $(-\hat{b}_1 - T)$, caused by ignoring the FX adjustment is economically significant in all cases. For example, for two-year strips the estimated FX-adjusted duration ranges from 2.48 years for Canada to 5.96 years for Denmark. This means that the adjusted duration is 25 to almost 200% higher than its Macaulay counterpart. For all other countries the error is greater than 50% of the unadjusted duration.

For five-year strips, the estimated FX-adjusted duration ranges from 5.73 years for Canada to 10.91 years for Denmark. Put another way, the adjusted duration is 15%-118% higher than Macaulay duration. For 10-year strips, the estimated FX-adjusted duration ranges from 10.7 years for Great Britain to 19.54 years for Denmark. In general, it seems that the lowest error among the examined countries is for strips issued in Canada and

[9]This is because: $b_1 = \frac{\partial \ln(V_{d,T,t})}{\partial r_{d,T,t}} = \frac{1}{V_{d,T,t}}\frac{\partial V_{d,T,t}}{\partial r_{d,T,t}} = -D_{d,T}$.

[10]Following Fons (1990), we estimate equation (8) with OLS.

Exhibit 1. Estimating the FX-Adjusted Duration

$$\ln(V_{d,T,t}) = a_1 + b_1\, r_{d,T,t} + e_{1t},$$

where $V_{d,T,t}$ is the value of a T-year foreign government strip on day t, converted to U.S. dollars, $r_{d,T,t}$ is the continuously compounded yield on a T-year U.S. Treasury strip index at time t, a_1, b_1, and e_{1t} are the regression intercept, slope, and error term, respectively. t-values are in parentheses. In addition to the estimates of the regression parameters and the adjusted R^2, the table reports the following: $SE(b_1)$ is the standard error of the slope estimator, and $t(-b_1-T)$ is the t-statistic used to test the hypothesis that the FX-adjusted duration is equal to T, n is the number of daily observations. Note that $-b_1$ is the estimate of the FX-adjusted elasticity because:

$$b_1 = \frac{\partial \ln(V_{d,T,t})}{\partial r_{d,T,t}} = \frac{1}{V_{d,T,t}} \frac{\partial V_{d,T,t}}{\partial r_{d,T,t}}.$$

	Australia					
Maturity (*T*)	***a*₁**	***b*₁**	***Adj. R*²**	***SE(b*₁)**	***t(-b*₁*-T)***	***n***
2	−0.4782	−3.3686	0.12	0.2382	5.7459	1408
	(−55.28)	(−14.14)				
5	−0.4869	−6.6258	0.23	0.3209	5.0664	1408
	(−36.26)	(−20.65)				
10	−0.4308	−13.2915	0.36	0.4710	6.9877	1408
	(−19.58)	(−28.22)				

	Canada					
Maturity (*T*)	***a*₁**	***b*₁**	***Adj. R*²**	***SE(b*₁)**	***t(-b*₁*-T)***	***n***
2	−0.3572	−2.4807	0.15	0.1543	3.1159	1426
	(−63.80)	(−16.08)				
5	−0.3547	−5.7298	0.30	0.2328	3.1354	1426
	(−36.42)	(−24.62)				
10	−0.3456	−11.1887	0.39	0.3711	3.2029	1426
	(−19.93)	(−30.15)				

	Denmark					
Maturity (*T*)	***a*₁**	***b*₁**	***Adj. R*²**	***SE(b*₁)**	***t(-b*₁*-T)***	***n***
2	−1.8453	−5.9584	0.31	0.2362	16.7576	1414
	(−214.93)	(−25.22)				
5	−1.7350	−10.9119	0.43	0.3366	17.5650	1414
	(−123.14)	(−32.42)				
10	−1.5315	−19.5384	0.52	0.5013	19.0279	1414
	(−65.39)	(−38.98)				

	Great Britain					
Maturity (*T*)	***a*₁**	***b*₁**	***Adj. R*²**	***SE(b*₁)**	***t(-b*₁*-T)***	***n***
2	0.4861	−3.2579	0.23	0.1564	8.0447	1425
	(85.70)	(−20.84)				
5	0.4854	−6.2458	0.38	0.2123	5.8674	1425
	(54.66)	(−29.42)				
10	0.4860	−10.5697	0.48	0.2940	1.9379	1425
	(35.40)	(−35.96)				

	Euro Countries					
Maturity (*T*)	***a*₁**	***b*₁**	***Adj. R*²**	***SE(b*₁)**	***t(-b*₁*-T)***	***n***
2	0.1616	−5.7553	0.30	0.2324	16.1587	1407
	(19.18)	(−24.76)				
5	0.2665	−10.4295	0.42	0.3286	16.5231	1407
	(19.40)	(−31.74)				
10	0.4618	−18.7205	0.51	0.4901	17.7929	1407
	(20.18)	(−38.20)				

Great Britain, while the highest error is for Danish and Euro strips. While the percentage error provides useful perspective, formally, the actual magnitude of the error is the relevant measure. For example, the errors for Canada are 1.48, 0.73, and 1.19 years for 2-, 5-, and 10-year strips, respectively. At the same time the errors for Great Britain are 3.76, 5.43, and 8.72 years for the same maturity strips, respectively. Thus, regression model (8) clearly demonstrates the need to adjust duration for FX risk.

Further, it is interesting to note that the adjusted R^2 increases with maturity for all countries, which implies that longer US yields explain more of the variability in the value of longer foreign strips than shorter US Treasury yields do for shorter foreign strips. The adjusted R^2's are the lowest for Australia where they range between 12%–36%, and the highest are for Denmark where they range between 31%-52%.

The results of regression model (8) demonstrate that the value of a US dollar based investor's position in a foreign bond is elastic with respect to US Treasury rates. From equations (6) and (9), one can express the FX-adjusted duration of the US dollar value of a foreign government strip as follows:

$$D_d = \frac{\partial r_f}{\partial r_d}T + \left(-\frac{1}{\phi_0}\frac{\partial \phi_0}{\partial r_d}\right). \qquad (12)$$

This implies that FX-adjusted elasticity could have two sources: (i) sensitivity of foreign government yields to changes in US Treasury yields $\left(\frac{\partial r_f}{\partial r_d}T \neq 0\right)$, and (ii) sensitivity of the US dollar spot FX rate for each country to US Treasury rates $\left(-\frac{1}{\phi_0}\frac{\partial \phi_0}{\partial r_d} \neq 0\right)$. We estimate the magnitude and measure the relative importance of each of these two components with regression models (10) and (11). Regression model (10) yields an estimator for the elasticity of the foreign bond, denominated in the foreign currency, with respect to the Treasury rate, while regression model (11) gives the elasticity of the spot FX rate with respect to the US Treasury rate. The results are reported in Exhibit 2.

The exhibit reports that the two components of the FX-adjusted elasticity are highly significant for all countries. The results of regression model (10) indicate that, $-\hat{b}_2$, which estimates the elasticity of the foreign strip denominated in the foreign currency, with respect to the US Treasury rate, is always significantly lower than the strip's unadjusted Macaulay duration (T) at a 95% confidence level based on t-ratios for $(-\hat{b}_2 - T)$.[11] The regressions also show that $-\hat{b}_2$, is significantly different from the estimated FX-adjusted elasticity $(-\hat{b}_1)$. This also implies that the interest rate policy ratio is less than unity: $\frac{\partial r_f}{\partial r_d} < 1$. Put another way, changes in foreign interest rates do not adjust 1:1 with respect to changes in US rates. At the same time, the adjusted R^2's of this regression model indicate that shifts in US Treasury rates explain a large portion of variability in the value of foreign strips, denominated in the foreign currency. In other words, the US Treasury yield is an important determinant in explaining the value, and therefore yield of a foreign government bond. For example, the adjusted R^2's of regression model (10) for Canada are 0.82, 0.81, and 0.76 for 2-, 5-, and 10-year strips, respectively.

Regression model (11) estimates $-\hat{b}_3$, the elasticity of the US dollar spot FX rate with respect to the US Treasury rate. The results indicate that the FX rate elasticity, $D_{\phi_0} = -\frac{1}{\phi_0}\frac{\partial \phi_0}{\partial r_d}$, is always positive for all countries and all maturities. This result is intuitive, because the US dollar strengthens as US Treasury rates increase, and therefore: $\frac{\partial \phi_0}{\partial r_d} < 0$. Note that, as expected (and by definition), it is always the case that the two estimated components are equal to the FX-adjusted elasticity estimated in regression model (8): $\hat{b}_1 = \hat{b}_2 + \hat{b}_3$.

Examining the relative magnitude of the two components, we note that the elasticity of the foreign strip component represents a higher ratio of the total FX-adjusted elasticity as compared with the FX rate elasticity component. For 2, 5, and 10-year Canadian strips, for example, the foreign strip elasticity component is 50.30%, 58.08%, and 59.24% of the total FX-adjusted duration, respectively. As suggested earlier, given the high sensitivity of Canadian government yields to US Treasury yields this result is expected. For all other countries, the FX rate elasticity is the dominant component. For example, for the Euro countries, the FX rate elasticity component is 82.52%, 72.67%, and 65.60% of the total FX-adjusted duration, for 2, 5, and 10-year Euro strips, respectively. It is also interesting to note that, in general, for all countries, as the maturity increases the relative magnitude of the foreign strip elasticity increases as well.

C. Implications for Portfolio Management

Our theoretical analysis shows how the calculation of duration needs to be adjusted to capture the interest rate sensitivity of default-free foreign sovereign bonds. Managers who include such bonds in executing unhedged, active rate-anticipation strategies should use our measure in assessing the sensitivity of their positions to shifts in foreign interest rates. In a similar vein, investors following passive immunization strategies must target our foreign-exchange adjusted duration in order to achieve truly immunized positions. Shifting to a forensic

[11]Due to space limitations in Exhibit 2 we do not report the individual t-ratios.

Exhibit 2. Estimating the Components of the FX-Adjusted Duration

$\ln(V_{f,T,t}) = a_2 + b_2\, r_{d,T,t} + e_{2t}$, and
$\ln(\phi_{0,t}) = a_3 + b_3\, r_{d,T,t} + e_{3t}$,

where $V_{f,T,t}$ is the value of a T-year foreign government strip on day t, denominated in the currency of the issuing government, $\phi_{0,t}$ is the spot FX rate on day t, a_2 and a_3 are intercepts, b_2 and b_3 are slope coefficients, and e_{2t} and e_{3t} are regression error terms. t-values are in parentheses. In addition to the estimates of the regression parameters and the adjusted R^2, the table reports the following: b_2/b_1 and b_3/b_1 represent the ratio of the interest-rate component and that of the exchange-rate component relative to the FX-adjusted duration; $b_2 + b_3$ is the sum of the estimated components (which must always equal), and n is the number of daily observations. Note that $-\hat{b}_2$ and $-\hat{b}_3$ are the estimates of the two components of the FX-adjusted elasticity because:

$$b_2 = \frac{\partial \ln(V_{f,T,t})}{\partial r_{d,T,t}} = \frac{1}{V_{f,T,t}}\frac{\partial V_{f,T,t}}{\partial r_{d,T,t}} \text{ and } b_3 = \frac{\partial \ln(\phi_{0,t})}{\partial r_{d,T,t}} = \frac{1}{\phi_{0,t}}\frac{\partial \phi_{0,t}}{\partial r_{d,T,t}}.$$

	Australia									
Maturity (T)	a_2	b_2	Adj. R^2	b_2/b_1	a_3	b_3	Adj. R^2	b_3/b_1	$b_2 + b_3$	n
2	−0.0848	−0.5125	0.54	15.21%	−0.3934	−2.8562	0.09	84.79%	−3.3686	1408
	(−186.18)	(−40.86)			(−17.97)	(−224.97)				
5	−0.1964	−1.7557	0.61	26.50%	−0.2905	−4.8701	0.13	73.50%	−6.6258	1408
	(−125.13)	(−46.82)			(−20.67)	(−14.50)				
10	−0.3343	−4.8233	0.70	36.29%	−0.0965	−8.4682	0.18	63.71%	−13.2915	1408
	(−85.89)	(−57.89)			(−4.27)	(−17.48)				

	Canada									
Maturity (T)	a_2	b_2	Adj. R^2	b_2/b_1	a_3	b_3	Adj. R^2	b_3/b_1	$b_2 + b_3$	n
2	−0.0359	−1.2479	0.82	50.30%	−0.3213	−1.2328	0.05	49.70%	−2.4807	1426
	(−63.43)	(−80.02)			(−61.23)	(−8.53)				
5	−0.0899	−3.3280	0.81	58.08%	−0.2648	−2.4018	0.09	41.92%	−5.7298	1426
	(−49.59)	(−76.80)			(−31.88)	(−12.10)				
10	−0.1942	−6.6283	0.76	59.24%	−0.1514	−4.5603	0.15	40.76%	−11.1887	1426
	(−41.96)	(−66.90)			(−11.37)	(−16.01)				

	Denmark									
Maturity (T)	a_2	b_2	Adj. R^2	b_2/b_1	a_3	b_3	Adj. R^2	b_3/b_1	$b_2 + b_3$	n
2	−0.0359	−1.1336	0.63	19.03%	−1.8094	−4.8248	0.26	80.97%	−5.9584	1414
	(−42.40)	(−48.67)			(−231.85)	(−22.47)				
5	−0.0767	−3.2445	0.69	29.73%	−1.6583	−7.6675	0.34	70.27%	−10.9119	1414
	(−31.45)	(−55.65)			(−138.26)	(−26.76)				
10	−0.1350	−7.1514	0.71	36.60%	−1.3965	−12.3870	0.40	63.40%	−19.5384	1414
	(−23.89)	(−59.14)			(−73.89)	(−30.62)				

	Great Britain									
Maturity (T)	a_2	b_2	Adj. R^2	b_2/b_1	a_3	b_3	Adj. R^2	b_3/b_1	$b_2 + b_3$	n
2	−0.0631	−0.9013	0.86	27.66%	0.5492	−2.3567	0.13	72.34%	−3.2579	1425
	(−180.12)	(−93.26)			(95.90)	(−14.93)				
5	−0.1467	−2.2781	0.84	36.47%	0.6320	−3.9677	0.19	63.53%	−6.2458	1425
	(−135.01)	(−87.70)			(70.48)	(−18.50)				
10	−0.2975	−3.8149	0.72	36.09%	0.7834	−6.7547	0.25	63.91%	−10.5697	1425
	(−102.02)	(−61.10)			(54.86)	(−22.09)				

	Euro Countries									
Maturity (T)	a_2	b_2	Adj. R^2	b_2/b_1	a_3	b_3	Adj. R^2	b_3/b_1	$b_2 + b_3$	n
2	−0.0335	−1.0058	0.68	17.48%	0.1951	−4.7495	0.26	82.52%	−5.7553	1407
	(−50.22)	(−54.73)			(24.95)	(−22.02)				
5	−0.0786	−2.8503	0.72	27.33%	0.3451	−7.5792	0.33	72.67%	−10.4295	1407
	(−39.36)	(−59.64)			(28.68)	(−26.33)				
10	−0.1439	−6.4392	0.73	34.40%	0.6056	−12.2813	0.39	65.60%	−18.7205	1407
	(−29.67)	(−61.99)			(31.94)	(−30.24)				

perspective, failure to make the recommended adjustment will lead to miscalculations of risk exposure for both types of investors. Our empirical tests demonstrate that such miscalculations would have been both statistically and economically significant across a range of foreign issuers. For foreign issuers in currencies whose monetary authorities follow policies independent of the US Federal Reserve (the European Community, for example) the miscalculation would have been particularly severe.

IV. Summary and Conclusions

This paper derives a model for the FX risk adjusted elasticity (duration) of foreign government bonds. Given the increasing investment in foreign sovereign debt, it is important to examine how the widely-used tool, duration, applies to such debt. We demonstrate that the FX risk adjusted duration has two components: the elasticity of the foreign bond denominated in the foreign currency and the elasticity of the FX rate with respect to the domestic currency. The reference for both elasticities is the domestic government yield. We further demonstrate that the FX adjustment will be trivial, and one can use the ordinary elasticity only when the following two conditions hold: (1) the foreign country conducts a 1:1 adjustment of its interest rate relative to changes in the domestic term structure, and (2) the spot rate is inelastic with respect to shifts in the domestic interest rate.

The implication of our model is that domestic managers of foreign bond portfolios who fail to adjust for FX risk engage in inefficient rate-anticipation and immunization strategies and may suffer substantial losses.

In the empirical section of the paper we look at the problem from the perspective of a US based investor who invests in bonds issued by various foreign governments. We estimate the FX-adjusted elasticity for several countries and different maturities. Our empirical results support the theory and indicate that the FX-adjusted elasticity is significantly different from the Macaulay type unadjusted elasticity for all countries and all maturities. The adjustment is both statistically and economically significant.

Decomposing the empirical FX-adjusted elasticity into the two components suggested by our theory we show that, in general, for most countries the elasticity of the FX rate dominates the foreign bond elasticity (the US Treasury yield is the reference rate for both). This finding is in agreement with Chow, Lee, and Solt (1997) who find that the impact of changes in exchange rates on bond returns is mainly driven by the correlation between exchange rate and interest rate changes.

The analysis in this paper assumes that sovereign risk is zero and that the foreign currencies of the bond issuers are fully convertible with efficient forward markets. Useful extensions in future research will address relaxing these assumptions to make the analysis relevant for important issuers like China, for example.

In brief, empirical results presented in our paper strongly suggest that the FX adjustment required for duration of foreign bonds is both statistically significant and economically nontrivial for a US investor. Ignoring this adjustment and using an unadjusted Macaulay duration instead may lead to substantial losses.■

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