

STAT 4015 Q

Midterm, July 27, 2009, 6:15pm - 7:50pm

This midterm is a closed book exam. Answer **four** out of the five problems. Mark clearly the problems you submit for scoring. You may provide answers in a formula form. Numerical evaluations are not required, but the use of a particular formula should be given a short and precise justification. Equal weights are given to all the five problems. Equal weights are given to all questions within each problem.

Problem 1. How many subsets of size 6 of the set $S = \{20, 21, \dots, 40\}$ contains:

- (1) At least one even number.
- (2) At least two even numbers.
- (3) Exactly 4 even numbers.

Problem 2. A customer that enters a vegetable store will buy tomatoes with probability of 22%, cucumbers with probability of 30%, and onion with probability of 28%. The customer will buy both tomatoes and cucumbers with probability of 11%, both tomatoes and onion with probability of 14%, both cucumbers and onions with probability of 10%, and all three vegetables with probability of 6%.

- (1) What is the probability that the customer will not buy any of the three vegetables?
- (2) What is the probability that the consumer will buy only one type of vegetable?
- (3) Given that the consumer buys only one type of vegetable, what is the probability that the vegetable is onion?

Problem 3. Six balls are randomly and sequentially selected (without replacement) from an urn that contains 15 black balls and 12 white balls.

- (1) What is the probability that the first and second balls are black and the fifth and sixth balls are white?
- (2) What is the probability of the event in (1), given that a total of 3 black balls and 3 white balls were selected?

Problem 4. Consider 3 dices. Dice A has 2 faces colored red and 4 faces colored green. Dice B has 3 faces colored red and 3 faces colored green. Dice C has 5 faces colored red and 1 face colored green. One of the dices is randomly selected and rolled three times.

- (1) What is the probability that a red face is obtained once and a green face is obtained twice?
- (2) Given that a red face is obtained once and a green face is obtained twice, what is the probability that the dice that was selected is Dice C?

Problem 5. A box contained 6 black balls and 4 white balls. A sample of three balls is randomly selected (without replacement). Let X be the difference between the number of black balls and the number of white balls in the sample.

- (1) Describe X as a random variable (i.e., the values it may obtain and the probabilities of these values).
- (2) What is the expectation of X ?
- (3) What is the variance of X ?