STAT 4015 Q

"Midterm", July 27, 2009, 6:15pm - 7:50pm

This midterm is a closed book exam. Answer **four** out of the five problems. Mark clearly the problems you submit for scoring. You may provide answers in a formula form. Numerical evaluations are not required, but the use of a particular formula should be given a short and precise justification. Equal weights are given to all the five problems. Equal weights are given to all questions within each problem.

Problem 1. A code is composes of 8 digits (0-9) and 5 letters (a-z, 26 in total). How many code combinations can be created if:

(1) The digits must come first and the letters must come last. Each letter can appear at most once.
(2) The first and the last positions must be letters, but there is no other restriction.
(3) Letters cannot be placed next to each other.

Problem 2. n people, including Erez and Rona, are randomly arranged in a line.

(1) What is the probability that the number of people that stand between them is at most 2 (i.e., it is any number between 0 and 2)?
(2) How would the answer change if the people are arranged in a circle?

Problem 3. Consider two urns. Urn A contains 5 green and 8 red balls and urn B contains 5 green and 10 red balls. At the first stage a ball is randomly taken from each of the urns. If both balls are of the same color then they are both placed in urn A and if they are of different colors then they are both placed in urn B. This completes the first stage. After the first stage an urn is randomly selected and a ball from that urn is selected.

(1) What is the probability that the ball that was selected in the second stage is red?
(2) Given that the ball that was selected in the second stage was red, what is the probability that the two balls from the first stage had different colors?

Problem 4. American roulette wheel has 38 numbers on it. Two gamblers are playing. One is always betting on the numbers 2, 4, and 6, and the other is always betting on the numbers 22 and 23. They will quite playing when either of them wins a bet.

(1) What is the probability that the first player is the winner?
(2) What is the probability that more than 20 round will be required before either of them win?
(3) Given the event in (2), what is the probability that the second player wins?

Problem 5. Let X be a random variable such that $\mathbb{E}(X) = 4$ and $\mathbb{E}[X(X-1)] = 12$

(1) Let $Y = 1.5X - 3$. What is the expectation and the variance of Y?
(2) Let $Z = aX + b$. What should be the values of a and b so that $\mathbb{E}(Z) = 0$ and $\mathbb{V}\mathrm{ar}(Z) = 1$?