MORE SOLUTIONS: CHAPTER 4, CLASS OF JULY 21

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Problems, 49. Correction to **part b** of the solution: Notice that we consider the probability space, conditional on the occurrence of even A. Consequently, the appropriate application of the Complete Probability formula is

$$\mathrm{P}(E|A) = \mathrm{P}(E|C_1, A)\mathrm{P}(C_1|A) + \mathrm{P}(E|C_2, A)\mathrm{P}(C_2|A) ,$$

where $\mathrm{P}(E|A, C_i)$ is defined to be $\mathrm{P}(E|A \cap C_i)$.

The question in the book refers to the first flip landing on heads and not the first three flips. Nonetheless, if we consider A to be the event that the first three flips all land on head then by Bayes formula

$$\mathrm{P}(C_1|A) = \frac{\mathrm{P}(A|C_1)\mathrm{P}(C_1)}{\mathrm{P}(A|C_1)\mathrm{P}(C_1) + \mathrm{P}(A|C_2)\mathrm{P}(C_2)} = \frac{(0.4)^3 \times 0.5}{(0.4)^3 \times 0.5 + (0.7)^3 \times 0.5} \approx 0.157 ,$$

which is not equal to 1/2. The problem as it appears in the 8th Edition of the book may be solved in a similar way.

Problems, 58. One may compare the probabilities of the events when they are computed under the assumption that the random variable satisfies $X \sim \mathrm{B}(n, p)$ to the case that they are computed under the assumption that the random variable has a Poisson distribution with the same mean, namely $X \sim \mathrm{Poisson}(np)$. We obtain:

(1) Binomial = 0.1488035, Poisson = 0.1437853;
(2) Binomial = 0.3151247, Poisson = 0.1300025;
(3) Binomial = 0.3486784, Poisson = 0.3678794;
(4) Binomial = 0.06606029, Poisson = 0.07230173.

Theoretical, 27. For a Geometric(p) random variable X we have that

$$\mathrm{P}(X > n) = \sum_{x=n+1}^{\infty} p(1-p)^{x-1} = (1-p)^n \sum_{x-n=1}^{\infty} p(1-p)^{(x-n)-1} = (1-p)^n .$$

(The fact that the final sum equals one may be validated by the substitution $x \to x - n$.) Now,

$$\mathrm{P}(X = x + n | X > n) = \frac{\mathrm{P}(X = x + n)}{\mathrm{P}(X > n)} = \frac{p(1-p)^{x+n-1}}{(1-p)^n} = p(1-p)^{x-1} .$$

It follows that the conditional distribution of $X - n$, given that $\{X > n\}$ is again Geometric(p). This property is given the name "lack of memory". If we think of paths in the grid of the random, once ones knows that after n steps one is still at level 0 then one may start the problem of determining how many steps are required to reach the level one anew, with the new zero set at n.

Theoretical, 28. Consider the grid of the random walk. The equality follows from the fact that one does not reach the level r after n steps if, and only if, the level at the n-th step is below r.