MORE SOLUTIONS: CHAPTER 8, CLASS OF AUGUST 6

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Problems, 13. Let $\bar{X}_{25}$ be the average score of the first class and let $\bar{Y}_{64}$ be the average score of the second class. We will be using the Central Limit Theorem in order to approximate the average score of each class.
(a) Notice that $\mathbb{E}(\bar{X}_{25}) = 74$ and $\mathrm{Var}(\bar{X}_{25}) = 14^2/25$.

$$\mathrm{P}\big(\bar{X}_{25} > 80\big) \approx \mathrm{P}\Big(Z > \frac{80-74}{14/5}\Big) = \mathrm{P}\big(Z > 2.143\big) = 0.016\ .$$

(b) For the second class again $\mathbb{E}(\bar{Y}_{64}) = 74$ but $\mathrm{Var}(\bar{Y}_{64}) = 14^2/64$.

$$\mathrm{P}\big(\bar{Y}_{64} > 80\big) \approx \mathrm{P}\Big(Z > \frac{80-74}{14/8}\Big) = \mathrm{P}\big(Z > 3.429\big) = 0.0003\ .$$

(c) Note that $\mathbb{E}(\bar{Y}_{64} - \bar{X}_{25}) = 0$. We will be making the dubious assumption that the average scores in the two classes are independent of each other. It will follow that but $\mathrm{Var}(\bar{Y}_{64} - \bar{X}_{25}) = 14^2/25 + 14^2/64 = 14^2 \cdot 0.234^2$.

$$\mathrm{P}\big(\bar{Y}_{64} - \bar{X}_{25} > 2.2\big) \approx \mathrm{P}\Big(Z > \frac{2.2}{14 \cdot 0.234}\Big) = \mathrm{P}\big(Z > 0.672\big) = 0.251\ .$$

(d) By the symmetry of the normal distribution:

$$\mathrm{P}\big(\bar{Y}_{64} - \bar{X}_{25} < -2.2\big) \approx \mathrm{P}\Big(Z < \frac{-2.2}{14 \cdot 0.234}\Big) = 0.251\ .$$

Problems, 16. Let S_{AJ} be the total processing time for A.J. and let S_{MJ} be the total processing time of M.J.. of the second class. We will be using the Central Limit Theorem in order to approximate the distributions of these sums.
(a) Notice that $\mathbb{E}(S_{\mathrm{AJ}}) = 1000$ and $\mathrm{Var}(S_{\mathrm{AJ}}) = 20 \cdot 10^2 = 2000$.

$$\mathrm{P}\big(S_{\mathrm{AJ}} < 900\big) \approx \mathrm{P}\Big(Z < \frac{900-1000}{\sqrt{2000}}\Big) = \mathrm{P}\big(Z < -2.236\big) = 0.0127\ .$$

(b) Notice that $\mathbb{E}(S_{\mathrm{MJ}}) = 1040$ and $\mathrm{Var}(S_{\mathrm{MJ}}) = 20 \cdot 15^2 = 4500$.

$$\mathrm{P}\big(S_{\mathrm{MJ}} < 900\big) \approx \mathrm{P}\Big(Z < \frac{900-1040}{\sqrt{4500}}\Big) = \mathrm{P}\big(Z < -2.087\big) = 0.0184\ .$$

(c) Note that $\mathbb{E}(S_{\mathrm{AJ}} - S_{\mathrm{MJ}}) = -40$ and, assuming independence, $\mathrm{Var}(S_{\mathrm{AJ}} - S_{\mathrm{MJ}}) = 2000 + 4500 = 6500$.

$$\mathrm{P}\big(S_{\mathrm{AJ}} - S_{\mathrm{MJ}} > 0\big) \approx \mathrm{P}\Big(Z > \frac{40}{\sqrt{6500}}\Big) = \mathrm{P}\big(Z > 0.496\big) = 0.310\ .$$

Theoretical, 8. The Gamma distribution with scale λ and shape t can be represented as a sum of n independent random variables, each with scale λ and shape t/n. If t is large and t/n converges to a positive constant then the Central Limit Theorem can be applied to the sum in order to argue proximity to normality.