An Empirical Study of Response and Sampling Errors for Multiplicity Estimates with Different Counting Rules

Gad Nathan


Stable URL: http://links.jstor.org/sici?sici=0162-1459%28197612%2971%3A356%3C808%3AAESORA%3E2.0.CO%3B2-M


Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/astata.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.
An Empirical Study of Response and Sampling Errors for Multiplicity Estimates with Different Counting Rules

GAD NATHAN*

The components of the mean square error (MSE) of estimates from surveys with multiplicity, where households report on events which occurred in them and also in other households (linked to them according to well-specified counting rules), can be estimated on the basis of a specially designed evaluation survey, under certain simplifying assumptions. The empirical results from such a survey, designed to estimate births and marriages, indicate that a wide counting rule is more efficient than a conventional rule, since the slightly higher response bias and variance are more than offset by the reduction in sampling variance.

1. INTRODUCTION

In the conventional retrospective household sample survey, used to estimate frequencies of demographic events, each demographic event is linked to a unique household, so that a report on each event is obtained from, at most, one household, if the linked household is in the sample. A short recall period, say a year, is usually used to reduce response errors due to faulty recall. This, however, results in high sampling errors (or alternatively the necessity for large sample sizes) due to the relatively low frequencies of demographic events in a short recall period. In an attempt to reduce sampling errors the use of surveys with multiplicity has been advocated by Sirken [2].

In a survey with multiplicity each demographic event may be linked to more than one household by a well-specified counting rule, e.g., births are linked to the households of the mother, of the mother's mother and of the mother's sisters. Each household in the sample is then asked to report both on all events to which it is linked and on the multiplicity of each event, i.e., the number of households to which the event is linked. Assuming that each event is linked to at least one household; that all events to which a household is linked are correctly reported by it; and that the multiplicity of each event is correctly reported, an unbiased estimate of the frequency of events of a given type can be obtained from a probability sample of households by weighting each report by the reciprocal of its multiplicity.

Sirken [3] has shown that multiplicity estimates with wide counting rules will, in general, reduce the sampling variance, in some cases considerably, as compared with the conventional estimate. Furthermore, empirical evidence has indicated that for deaths, coverage biases resulting from incomplete reporting may also be lower for multiplicity surveys than for conventional surveys [4]. However, in general, higher response errors might be expected from reporting by more distant relatives, with differential biases according to the counting rule. Thus, in order to compare different counting rules (including a comparison of multiplicity rules with the conventional survey), it is necessary to consider and to evaluate the total mean square error (mse) and its components, including sampling and response errors.

This is done in Sections 2 and 3 by a decomposition of the total mse of the estimate, according to standard models, into components of response bias, response variance and sampling variance (assuming no response-sampling interactions), which are evaluated separately.

The response bias, as pointed out in [3], is composed of a counting rule bias and of an implementation bias. The counting rule bias is the bias component which would remain for a full census with perfect reporting and results from events not linked under the counting rule to any household. While this may be considerable for deaths, it will, in general, be negligible for births and marriages, even for the conventional counting rule, e.g., cases of death or emigration of the mother since the event, and is not considered in the present model.

In Section 4, it is shown that, on the basis of a survey with multiplicities, for which a subsample of reported relatives is surveyed according to the same counting rules, it is possible to estimate the various components of mse, under certain simplifying assumptions.

The proposed model was tested on an experimental survey, with an evaluation survey, conducted by the Central Bureau of Statistics, which is described in Section 5. The basic survey was carried out on an equal-prob-

* Gad Nathan is Senior Lecturer, Department of Statistics, Hebrew University, Jerusalem, Israel, and is in charge of planning and development, Central Bureau of Statistics, Jerusalem, Israel. This paper is a revised version of a contributed paper presented at the second session of the International Association of Survey Statisticians, Warsaw, September 1975. Research was partially supported by a research agreement with the U.S. National Center for Health Statistics (No. 06-698-R) and by a grant from the United States-Israel Binational Foundation, (Project No. 462). The author is grateful for the help of Uziel O. Schmelz and Jay Kenin in the joint execution of the research agreement, and for the many helpful comments of Monroe G. Sirken.
ability sample of some 4,000 households, from whom reports were requested on births occurring during the calendar year 1973 not only to all ever-married women in the household but also on births to daughters and to sisters of women in the household who live in another household. Similarly, reports were requested on marriages of all members of the household, of their sons and daughters, and of their siblings. To estimate response errors, an evaluation survey was carried out according to the same procedure as that of the basic survey on a subsample of some 500 households reported as linked to the events reported in the basic survey.

On the basis of the theoretical model of Sections 2 to 4, with certain necessary modifications, the estimates of the components of total error were computed. They are presented in Section 6, separately for births and for marriages, for

1. the conventional rule; and for each of two multiplicity counting rules:
2. births linked to households of mother and of mother’s mother (marriages linked to households of the married person and of his/her parents);
3. births linked to households of mother, of mother’s mother and of mother’s sisters (marriages linked to households of married person, of his/her parents and of his/her siblings).

The results provide insights on the relative efficiencies of the three counting rules as measured by the overall error and its components. Thus, the wider multiplicity rule (3) is considerably more efficient for the sample size considered, than both the restricted multiplicity rule (2) and the conventional rule (1). The increase in efficiency is primarily due to the reduction in the variance components, where the larger response variance of the multiplicity rules is more than offset by their smaller sampling variances. Both undercoverage and overcoverage biases increase in general with the use of wider multiplicity counting rules. However, the net bias of the wider multiplicity rule (3) for marriages is even smaller than that for the conventional and is not much larger for births. The restricted multiplicity rule (2) has the largest net biases, both for births and for marriages. The results thus indicate the overall superiority of the full multiplicity rule for the range of sample sizes considered.

2. THE MODEL AND NOTATION

Let \( I_a(\alpha = 1, \ldots, N) \) be the events which occurred during the reference period. In addition, let \( I_a(\alpha = N + 1, \ldots, N + M) \) be \( M \) nonevents which are liable to be reported by households in the survey, e.g., events which occurred outside the reference period. Reports on events and nonevents are to be made by a simple random sample, \( S \), of size \( \ell \) selected without replacement out of a population of households, \( H_i(\ell = 1, \ldots, L) \), according to a well-specified counting rule. A counting rule is defined by a set of indicator variables \( \delta_{a,i,r} = 1 \) if \( I_a \) is linked to \( H_i \) via a relative of highest degree \( r \), and equals zero otherwise, where a hierarchy of degrees of relation, \( r \), is assumed, (e.g., for births—mother; mother’s mother; mother’s sisters); and by a set of degrees of relation, \( C \), such that

\[
\delta_{a,i} = \sum_{r \in C} \delta_{a,i,r} \leq 1
\]

For a given counting rule, the multiplicity of \( I_a \) for degree of relation \( r \) is assumed, (e.g., for births-...
and trials is
\[
N_0 = \text{E} \{ \text{E}_t(\hat{N}_S, t | S) \} = \text{E}_s(\hat{N}_S) = \sum_{i=1}^{L} X_i . \tag{2.5}
\]

Assuming no response-sampling interaction, the overall MSE of the estimate can be decomposed into a response variance component, a sampling variance component and a response bias component, respectively, as follows:
\[
\text{MSE}(\hat{N}_S, t) = \text{E}_s(\hat{N}_S, t - N)^2 = \text{E}_s[\text{E}_t(\hat{N}_S, t - \hat{N}_0)^2 | S] + \text{E}_s(\hat{N}_S - \hat{N}_0)^2 = RV + SV + B^2 . \tag{2.6}
\]

3. EVALUATION OF THE ERROR COMPONENTS

To evaluate the bias, \( B \), let the probability of (correctly) reporting an event that occurred, by a relative of degree \( r \), be
\[
P_{1r} = P(\varepsilon_{a,i,r}(t) = 1 | \delta_{a,i,r} = 1) ; \quad (\alpha = 1, \ldots, N) ,
\]
where it is assumed that this probability is independent of the event \( I_a \) and of the household \( H_r \). Similarly, let the probability of (incorrectly) reporting a nonevent, \( I_a(\alpha = N + 1, \ldots, N + M) \), by a relative of degree \( r \), be \( P_{or} \).

Denote the weighted contribution to the total number of true events of reports by relatives of degree \( r \), by \( \sum_{\alpha=1}^{N} s_{a,i,r} / s_a \) and, similarly, the weighted contribution to the total number of nonevents of reports by relatives of degree \( r \) by \( \sum_{\alpha=1}^{N+M} m_{a,i,r} / m_a \).

If the relative variance of \( s_{a,i}(t) \) is assumed to be independent of \( \alpha \) and of \( i \), so that
\[
\text{Var} \{ s_{a,i}(t) / s_a \} = V^2 ; \tag{3.1}
\]
for all \( \alpha \) and \( i \), then the bias can be approximated by
\[
B = N_0 - N \approx \sum_{r \in \mathbb{E}} \left\{ 1 - (1 + V^2) P_{1r} \right\} N_r + (1 + V^2) \sum_{r \in \mathbb{E}} P_{or} M_r , \tag{3.2}
\]
where the first term reflects undercoverage of event reporting and the second term reflects over-reporting of nonevents.

The response variance, \( RV \), is a function of the simple response variance and the correlated response variance
\[
RV = L^2 / \ell \{ SVR + (\ell - 1) CRV \} , \tag{3.3}
\]
where the simple response variance is defined as
\[
\text{SRV} = \frac{1}{L} \sum_{i=1}^{L} E_t(X_{it} - X_i)^2 , \tag{3.4}
\]
and the correlated response variance is defined as
\[
\text{CRV} = \left[ L(L - 1) \right]^{-1} \sum_{i \neq j} E_t(\hat{X}_{it} - \hat{X}_i)(\hat{X}_{jt} - \hat{X}_j) . \tag{3.5}
\]

Although the correlated response variance has a relatively high weight in (3.3), it will be neglected in the following. This is primarily because its estimation requires a much more complex evaluation survey design than that which is usually possible. In addition, it is believed that in this case the correlated component may indeed be extremely small since it seems that response deviations for reports on events are primarily due to pure respondent error than to enumerator effects.

Define the weighted average of squared reciprocals of multiplicities for reports by relatives of degree \( r \) for events by
\[
U_{1r} = \frac{1}{N_r} \sum_{a=1}^{N} \frac{\delta_{a,i,r}}{s_a} , \tag{3.6}
\]
and for nonevents by
\[
U_{or} = \frac{1}{M_r} \sum_{a=N+1}^{N+M} \frac{\delta_{a,i,r}}{s_a} . \tag{3.7}
\]
Then the simple response variance (3.4) can be approximated by
\[
\text{SRV} \approx L^{-1} \sum_{r \in \mathbb{E}} \left\{ N_r \left\{ U_{1r} P_{1r} V^2 + P_{1r}(1 - P_{1r})(1 + V^2) \right\} + M_r \left\{ U_{or} P_{or} V^2 + P_{or}(1 - P_{or})(1 + V^2) \right\} \right\} , \tag{3.8}
\]
under the assumption that \( \varepsilon_{a,i,r}(t) / s_a \) are uncorrelated for different events.

Similarly, the sampling variance can be approximated by
\[
\text{SV} \approx \left[ (L - \ell) / \ell \right]\left\{ (1 + V^2) \sum_{r \in \mathbb{E}} (P_{1r}^2 N_r U_{1r} + P_{or}^2 M_r U_{or}) + W \right\} - N_0^2 / L , \tag{3.9}
\]
where
\[
W = \sum_{\alpha=1}^{L} \sum_{r' \in \mathbb{E}} \left\{ \sum_{a,a'=1}^{N} P_{1r} P_{1r'} \frac{\delta_{a,i,r} \delta_{a',i,r'}}{s_a s_{a'}} + \sum_{a,a'=N+1}^{N+M} P_{or} P_{or'} \frac{\delta_{a,i,r} \delta_{a',i,r'}}{s_a s_{a'}} \right\} + 2 \sum_{\alpha=1}^{N} \sum_{a-N-1}^{N+M} P_{1r} P_{or'} \frac{\delta_{a,i,r} \delta_{a',i,r'}}{s_a s_{a'}} , \tag{3.10}
\]
and \( N_0 \) is given by (2.5).

4. ESTIMATION OF COMPONENTS FROM AN EVALUATION SURVEY

To estimate the components defined by (3.2), (3.8), and (3.9), estimates of the following parameters are required: \( P_{1r}, P_{or}, V^2, N_r, M_r, U_{1r}, U_{or} \), and \( W \).

Certain combinations of these parameters can be estimated from the multiplicity survey itself (henceforth "the basic survey"). Thus, define
\[
\hat{T}_{r,(1)} = L^{-1} \sum_{i=1}^{L} \delta_{a,i,r}(t) \sum_{a=1}^{N+M} \frac{\delta_{a,i,r}(t)}{s_a(t)} , \tag{4.1}
\]
so that \( \sum_{r \in \mathbb{E}} \hat{T}_{r,(1)} = \hat{N}_S, t \).

Then it is easy to see from Section 3 that
\[
E[\hat{T}_{r,(1)}] = (1 + V^2)(P_{1r} N_r + P_{or} M_r) . \tag{4.2}
\]
Similarly, if we define

$$\hat{T}_r^{(2)} = \frac{L}{l} \sum_{i=1}^{L} \sum_{a=1}^{N+M} \frac{d_i \varepsilon_{a,i,r}(t)}{s_{a,i}^2(t)},$$

(4.3)

then

$$E[\hat{T}_r^{(2)}] = (1 + V^2)[P_{1r}U_{1r}N_r + P_{0r}U_{0r}M_r].$$

(4.4)

However, for estimation of the individual parameters an evaluation survey is required.

The model for the design of the evaluation survey assumes that of the total number of events reported in the basic survey by relatives of degree $r$

$$m_r = \sum_{i=1}^{L} \sum_{a=1}^{N+M} \varepsilon_{a,i,r}(t),$$

(4.5)

a simple random sample without replacement of size $n_r$ is selected.

Let $\varepsilon_{a,i,r} = 1$, if $I_a$ is selected for evaluation on the basis of a report in the basic survey by $H_i$, via a relative of degree $r$, and equal zero otherwise, so that

$$n_r = \sum_{i=1}^{L} \sum_{a=1}^{N+M} \varepsilon_{a,i,r}.$$  

(4.6)

For each event thus selected for evaluation, one of the households of the multiplicity network in the basic survey is selected for an independent evaluation report, by the same method as in the basic survey, via a relative of different degree. For $r = 1$ (reports by the event household in the basic survey), $n_{1r}$ ($r' \neq 1$) evaluation events are randomly assigned for evaluation by relatives of degree $r'$. For $r > 1$ (reports in the basic survey by other relatives) the event household itself is selected for the evaluation survey.

It is assumed that for each evaluation report a determination can be made if it relates to a true event ($\alpha = 1, \ldots, N$) or to a nonevent ($\alpha = N + 1, \ldots, N + M$). Let $f_{a,r,r'} = 1$, if $\varepsilon_{a,i,r} = 1$ (for some $H_i$) and a relative of degree $r'$ is selected for evaluation, and equal zero otherwise, so that $\sum_{a=1}^{N+M} f_{a,r,r'} = n_{1r}$. Denote the result of evaluation by the indicator variable $g_{a,i,r} = 1$, if $f_{a,i,r} = 1$ and $I_a$ is reported in the evaluation survey (by a relative of degree $r'$), and $g_{a,r,r'} = 0$, otherwise.

For $r \neq 1$, it can easily be seen that $P_{1r}$ can be estimated by the proportion of correctly reported events by relatives of degree $r$

$$\hat{P}_{1r} = \sum_{a=1}^{N} g_{a,1,r} / \sum_{a=1}^{N} f_{a,1,r}, \quad (r \neq 1).$$

(4.7)

Using these estimates, $P_{11}$ can then be estimated by

$$\hat{P}_{11} = \left(\sum_{r \in C} \sum_{a=1}^{N} g_{a,r,1m} \hat{p}_{1r}\right) / \left(\sum_{r \in C} \sum_{a=1}^{N} f_{a,r,1m} \hat{p}_{1r}\right),$$

(4.8)

where $\hat{p}_{1r}$ is the number of households of degree $r$, as reported in the basic survey. Values of $P_{0r}$ can be estimated in exactly the same way by taking the summations in (4.7) and (4.8) over the values $\alpha = N + 1, \ldots, N + M$.

Consider next the ratio of the two components of the right side of (4.2):

$$Q_r^{(1)} = (P_{0r}M_r) / (P_{1r}N_r).$$

(4.9)

It can easily be seen that $Q_r^{(1)}$ is estimated by

$$\hat{Q}_r^{(1)} = \left(\sum_{i=1}^{L} \sum_{a=N+1}^{N+M} \frac{\varepsilon_{a,i,r}}{s_{a,i}^2(t)}\right) / \left(\sum_{i=1}^{L} \sum_{a=1}^{N} \frac{\varepsilon_{a,i,r}}{s_{a,i}^2(t)}\right).$$

(4.10)

Similarly, the ratio,

$$Q_r^{(3)} = (P_{0r}U_{0r}M_r) / (P_{1r}U_{1r}N_r),$$

(4.11)

can be estimated by

$$\hat{Q}_r^{(3)} = \left(\sum_{i=1}^{L} \sum_{a=N+1}^{N+M} \frac{\varepsilon_{a,i,r}}{s_{a,i}^2(t)}\right) / \left(\sum_{i=1}^{L} \sum_{a=1}^{N} \frac{\varepsilon_{a,i,r}}{s_{a,i}^2(t)}\right).$$

(4.12)

If for events selected for the evaluation study and reported by both households, $s_{a,i}(t)$ and $s_{a,i'}(t')$ are the reported multiplicities, then

$$V_{a,i}^2 = 2 \frac{[s_{a,i}(t) - s_{a,i'}(t')]^2}{[s_{a,i}(t) + s_{a,i'}(t')]^2},$$

(4.13)

has the approximate expectation $V^2$, so that their average,

$$V^2 = \left(\sum_{r, r'} \sum_{i=1}^{L} g_{a,r,r'} \frac{V_{a,i}^2}{1 + \hat{V}_{1r}}\right) / \left(\sum_{r, r'} \sum_{i=1}^{L} g_{a,r,r'}\right),$$

(4.14)

is an approximately unbiased estimate of $V^2$.

Finally, from (4.2) and (4.9), $N_r$ and $M_r$ can be estimated by

$$\hat{N}_r = \hat{P}_{1r}[1 + \hat{V}_r] \hat{P}_{1r} / (1 + \hat{Q}_r^{(1)}),$$

(4.15)

and

$$\hat{M}_r = \hat{P}_{1r} \hat{N}_r \hat{Q}_r^{(1)} / \hat{P}_{0r}.$$  

(4.16)

Similarly, $U_{1r}$ and $U_{0r}$ can be estimated by

$$\hat{U}_{1r} = \hat{T}_r^{(3)} / \hat{T}_r^{(3)} + \hat{Q}_r^{(1)},$$

(4.17)

and

$$\hat{U}_{0r} = \hat{U}_{1r} \hat{Q}_r^{(1)} / \hat{Q}_r^{(1)}.$$  

(4.18)

This completes the estimation of all parameters except $W$ (3.10). If the evaluation sample is large enough, the value of $W$ can be estimated from the corresponding evaluation sample values in essentially the same way as just described.

5. THE EXPERIMENTAL SURVEY AND ITS EVALUATION

The experimental survey was conducted by adding a special multiplicity questionnaire onto a selected part of the regular quarterly Israel Labor Force Survey in the first quarter of 1974. The population for the multiplicity survey was defined as all Jewish households, excluding those in institutions and in kibbutzim. The Labor Force Survey is a self-representing probability sample, selected
as a systematic random sample from apartment-tax lists in all large urban localities and in a stratified random sample of small localities. It is subdivided into panels and into enumeration-weeks, each part being a self-representing probability sample. For the multiplicity survey nine enumeration-weeks for three out of the four panels were selected. Since institutions and kibbutzim, where clustering effects may be high, were not included, the sample may be regarded, for practical purposes, as a simple random sample without replacement.

The basic Labor Force Survey questionnaire includes a listing of all household members with their demographic characteristics. The supplementary multiplicity questionnaire included, separately for births and for marriages, a screening questionnaire and an event report. In the screening questionnaire all ever-married women in the household were asked if they gave birth during the calendar year 1973 (and all adults if they married in 1973). Then, all ever-married women, aged 34 and above, were asked if they had daughters, outside the household, who gave birth in 1973 (and all ever-married persons, aged 34 and above, were asked if they had sons or daughters, outside the household, who married in 1973). Finally, all women in the household, whose mother did not reside there, were asked if they had sisters, outside the household, who gave birth in 1973 (and all persons, neither of whose parents resided in the household, were asked if they had siblings, outside the household, who married in 1973).

Only if an event occurred to a member of the household or to a specified relative, was it necessary to complete an event report. This included basic demographic data on the event (sex, month of occurrence, age of mother or of married person) and a listing of the multiplicity network—all specified relatives of the event person, i.e., mother and sisters of woman who gave birth; parents and siblings of person who married. The address of each listed relative was recorded, to obtain the multiplicity s, (t). Thus, for practically all Labor Force Survey questionnaires completed, a multiplicity questionnaire was also obtained (a total of 4,080). Although there was some differential noncompletion by district, the treatment of the sample as simple random should not considerably affect the comparisons of the relative efficiencies of the different counting rules.

The evaluation survey included 374 households of relatives reported in the basic survey, selected according to the design of Section 3. Thus, for reports of events occurring to members of the household in the basic survey addresses of their sisters (siblings), residing in different households were selected. For samples of events reported to offspring and of events reported to siblings in the basic survey, the event household was included in the evaluation survey. Incomplete addresses and some addresses in outlying localities were excluded.

The evaluation survey was conducted exactly according to the procedure of the basic survey (including the household listing), in April–May 1974. The two reports on the same event were compared and the following decision rules for assessment of under-reporting and over-reporting were used:

1. Events reported both in the basic survey and in the evaluation survey were considered as correctly reported true events, i.e., a 0, 1 ≤ a ≤ N.
2. The status of events not reported in the evaluation survey was adjudicated on the basis of the Labor Force Survey Listing form of the event household (mother's household for births; married person's household for marriages), as follows:
   2.1 If the event was not listed or was listed for the wrong year, it was considered as a nonevent, over-reported by the basic survey household which reported it, i.e., N + 1 ≤ α ≤ N + M and a 0, 1 ≤ a ≤ N.
   2.2 If the event was listed in the correct year, it was considered as a true event, under-reported by the evaluation household, i.e., 1 ≤ a ≤ N and a 0, 1 ≤ a ≤ N + M.

Note that this enables the estimation of the values of P, by (4.7) and (4.8), but not of the values of P, since a 0 for all N + 1 ≤ a ≤ N + M. This is due to the fact that for nonevents the evaluation is used to determine that it is indeed a nonevent, so that no evaluation of the probability that the nonevent is reported is available. Nevertheless, all the remaining relevant parameters of Section 4 can be estimated by (4.14), (4.15), (4.17), and (4.18), with the exception of M, (4.18). However, the product P, M, can be estimated from (4.16) by P, N, Q,.

This in turn implies that all terms of the estimates of the bias (3.12), of the response variance (3.8) and of the sampling variance (3.9) can be estimated, with the exception of the terms \( \sum_{r \in C} P_{o, r}^2 M_r \) in (3.8) and of \( \sum_{r \in C} P_{o, r}^2 M_r U_o, \) in (3.9). These were estimated, by a slight modification of the model, as follows:

\[
\sum_{r \in C} P_{o, r}^2 M_r = \hat{U}_o \hat{M}_0
\]  
and
\[
\sum_{r \in C} P_{o, r}^2 M_r U_o = \hat{U}_o \hat{M}_0 ,
\]
where
\[
\hat{M}_0 = \sum_{r \in C} \hat{P}_{1, r} \hat{N}_r \hat{Q}_r(1)
\]  

The execution of the basic survey in difficult conditions, shortly after the 1973 (Arab-Israeli) war, resulted in a relatively low completion rate (81 percent). However, the bias (3.12), of the response variance (3.8) and of the sampling variance (3.9) can be estimated, with the exception of the terms \( \sum_{r \in C} P_{o, r}^2 M_r \) in (3.8) and of \( \sum_{r \in C} P_{o, r}^2 M_r U_o, \) in (3.9). These were estimated, by a slight modification of the model, as follows:
Response and Sampling Errors for Multiplicity Estimates

This can be shown to be equivalent to assuming that the probability of over-reporting, \( P_{\text{over}} \), is the average of the reciprocals of the multiplicities, i.e., that on the average only one household in the network over-reports the event. While no empirical evidence is available this assumption seems reasonable, and in any case the terms estimated under this assumption are small relative to the other terms in the variance components.

Finally, the term \( W \) (3.10) was estimated as zero, since only a very small number of households reported more than a single event.

6. ANALYSIS OF RESULTS

The main results are given in Table 1, separately for births and marriages, for each of the three counting rules considered. The current demographic estimates are based on virtually complete vital registration for the calendar year 1973, with approximate adjustments for the survey population. The remainder of the results of Table 1 were estimated on the basis of the model of Section 4 with the modifications described at the end of Section 5.

Thus, the basic survey estimate is \( N_{s,t} \) (2.3). The estimate of the net bias is the expression (3.2), with the parameters estimated according to Section 4. The revised survey estimate is then obtained by subtracting the estimated bias from the basic survey estimate. The estimate of net bias is further divided into estimates of the undercoverage bias—the first term of (3.2)—and the overcoverage bias—the second term of (3.2). The estimate of the total MSE (2.6) is broken down into its components: the squared bias, the sampling variance and the response variance. Finally relative standard errors are given, as percentages of the revised survey estimate.

A first glance at Table 1 immediately draws attention to the considerable differences between the current demographic estimates and the survey estimates. Both the basic survey estimate and the revised survey estimate are systematically far below the demographic estimates, for all three counting rules, both for births and for marriages. However, the difference between the revised survey estimate and the demographic estimate are covered by two standard errors, for the conventional and for the restricted multiplicity rules, and only barely exceed two standard errors, for the full multiplicity rule.

Furthermore, the estimates for the different counting rules are not independent. Thus, the differences between the demographic estimates and the survey estimates may be due to a systematic sampling bias (because of non-completion both in the basic survey and in the evaluation survey), or to a systematic counting rule bias, e.g., as a result of events for which all possible reporters have died, emigrated or are otherwise outside the survey population. Neither of these biases were taken into account in the model. However, comparisons of the survey distributions, by demographic characteristics, with those of the national demographic estimates did not indicate any striking differences, and it is unlikely that either of the two types of bias could be considerable. Furthermore, the fact that the differences are similar for the different counting rules

<table>
<thead>
<tr>
<th>Component</th>
<th>Current demographic estimate</th>
<th>Revised survey estimate</th>
<th>Basic survey estimate</th>
<th>Estimate of net bias</th>
<th>Overcoverage bias</th>
<th>Undercoverage bias</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Births</td>
<td>Marriages</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>Restricted multiplicity</td>
<td>Full multiplicity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current demographic estimate</td>
<td>64,250</td>
<td>64,250</td>
<td>64,250</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revised survey estimate</td>
<td>59,700</td>
<td>60,526</td>
<td>58,016</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic survey estimate</td>
<td>60,016</td>
<td>62,305</td>
<td>58,925</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate of net bias</td>
<td>+48</td>
<td>+1,780</td>
<td>+935</td>
<td>-1,214</td>
<td>-1,214</td>
<td>-1,214</td>
</tr>
<tr>
<td>-Undercoverage bias</td>
<td>-1,223</td>
<td>-2,857</td>
<td>-4,766</td>
<td>-1,214</td>
<td>-1,214</td>
<td>-1,214</td>
</tr>
<tr>
<td>-Overcoverage bias</td>
<td>+1,271</td>
<td>+4,837</td>
<td>+5,701</td>
<td>0</td>
<td>+980</td>
<td>+980</td>
</tr>
<tr>
<td>Components of MSE (×10⁻³)</td>
<td>10,131</td>
<td>10,106</td>
<td>6,274</td>
<td>8,997</td>
<td>18,365</td>
<td>5,485</td>
</tr>
<tr>
<td>Squared bias</td>
<td>2</td>
<td>3,168</td>
<td>874</td>
<td>1,474</td>
<td>12,557</td>
<td>1,141</td>
</tr>
<tr>
<td>Total variance</td>
<td>10,129</td>
<td>6,938</td>
<td>5,400</td>
<td>7,523</td>
<td>5,808</td>
<td>4,344</td>
</tr>
<tr>
<td>Sampling variance</td>
<td>9,825</td>
<td>5,900</td>
<td>3,815</td>
<td>7,307</td>
<td>4,969</td>
<td>3,154</td>
</tr>
<tr>
<td>Response variance</td>
<td>303</td>
<td>1,038</td>
<td>1,584</td>
<td>216</td>
<td>839</td>
<td>1,190</td>
</tr>
<tr>
<td>Relative standard errors (percentages)</td>
<td>5.31</td>
<td>5.25</td>
<td>4.32</td>
<td>6.65</td>
<td>10.02</td>
<td>5.51</td>
</tr>
<tr>
<td>Bias (absolute value)</td>
<td>0.08</td>
<td>2.94</td>
<td>1.61</td>
<td>2.69</td>
<td>8.28</td>
<td>2.51</td>
</tr>
<tr>
<td>Sampling standard error</td>
<td>5.23</td>
<td>4.01</td>
<td>3.37</td>
<td>5.99</td>
<td>5.21</td>
<td>4.18</td>
</tr>
<tr>
<td>Response standard error</td>
<td>0.92</td>
<td>1.68</td>
<td>2.17</td>
<td>1.03</td>
<td>2.14</td>
<td>2.57</td>
</tr>
</tbody>
</table>
2. Analysis of Relative Biases by Reporting Relatives and Counting Rule (Percentages)

<table>
<thead>
<tr>
<th>Category</th>
<th>Under-coverage bias (Births)</th>
<th>Over-coverage bias (Births)</th>
<th>Net bias (Births)</th>
<th>Under-coverage bias (Marriages)</th>
<th>Over-coverage bias (Marriages)</th>
<th>Net bias (Marriages)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Event household</strong></td>
<td>-2.04</td>
<td>+2.12</td>
<td>+0.08</td>
<td>-2.69</td>
<td>0</td>
<td>-2.69</td>
</tr>
<tr>
<td><strong>Household of parent</strong></td>
<td>-10.34</td>
<td>+16.20</td>
<td>+5.86</td>
<td>-9.76</td>
<td>+36.96</td>
<td>+27.20</td>
</tr>
<tr>
<td><strong>Household of sibling</strong></td>
<td>-20.51</td>
<td>+15.73</td>
<td>-4.78</td>
<td>-21.74</td>
<td>+7.41</td>
<td>-14.33</td>
</tr>
</tbody>
</table>

### Counting rule

**Conventional**

| Total bias                 | -2.04                       | +2.12                       | +0.08            | -2.69                      | 0                           | -2.69               |

**Restricted multiplicity**

| Total event-reporting bias | -5.28                       | +7.62                       | +2.34            | -4.88                      | +11.46                     | +6.58               |
| Contribution of multiplicity reporting variance | +0.56                       | +0.04                       | +0.60            | +1.52                      | +0.18                       | +1.70               |
| Total bias                 | -4.72                       | +7.66                       | +2.94            | -3.36                      | +11.64                     | +8.28               |

**Full multiplicity**

| Total event-reporting bias | -9.14                       | +9.73                       | +0.59            | -9.81                      | +10.86                     | +1.05               |
| Contribution of multiplicity reporting variance | +0.92                       | +0.10                       | +1.02            | +1.30                      | +0.16                       | +1.46               |
| Total bias                 | -8.22                       | +9.83                       | +1.61            | -8.51                      | +11.02                     | +2.51               |

(though slightly larger for the full multiplicity rule), indicates that the biases not considered in the model should not affect very much the comparisons of overall errors between the counting rules.

An examination of the total MSE's shows that the overall errors for the full multiplicity rule are considerably lower than for the conventional rule, both for births and for marriages (for the sample size of the survey—4,084). However, for the restricted multiplicity rule a considerably larger MSE than for the conventional rule is obtained for marriages (due to an extremely high bias—over 8 percent—for this counting rule), while for births it is close to that of the conventional rule. The relative root mean square errors for births (4.3-5.3 percent) are lower than the errors for marriages (5.5-10.0 percent), primarily because of lower biases.

The counting rules differ considerably in the composition of the MSE, especially with respect to the bias and variance components. Thus, the variance (sampling and response) accounts for practically all the MSE for births with the conventional rule, but accounts for only about 30 percent for marriages with the restricted multiplicity rule. Hence, each of the components must be considered separately as is done in the following. In addition, this implies that the relationships between the overall errors for the different counting rules hold only for the sample size of this survey and may differ considerably for other sample sizes, since sample size affects only the variance component.

As could be expected, the net biases are in general larger for the multiplicity counting rules than for the conventional rule. Thus, for births it is close to zero for the conventional rule and goes up to +2.9 percent for the restricted multiplicity rule. For marriages it ranges from -2.7 percent, for the conventional rule up to +8.3 percent for the restricted multiplicity rule. However, in absolute value the net bias for the full multiplicity rule for marriages is somewhat less than that for the conventional. The differences are explained by the analysis of the relative biases into their components as given in Table 2.

Table 2 shows that undercoverage biases increase consistently with distance of relationship, similarly for births and for marriages, from 2.0-2.7 percent for event households, to 9.8-10.3 percent for parents and to 20.5-21.7 percent for siblings. This explains the consistent increase in total event-reporting undercoverage for wider counting rules—2.0-2.7 percent for conventional, 4.9-5.3 percent for the restricted multiplicity rule and 9.1-9.8 percent for the full multiplicity rule. However the contribution of the multiplicity reporting variance, which is positive for multiplicity counting rules, reduces these differences somewhat. Thus, the total undercoverage bias is reduced for the full multiplicity rule to 8.2-8.5 percent, and for the restricted multiplicity rule to 4.7 percent for births and to 3.4 percent for marriages.

The overcoverage biases also vary considerably by reporting relative. Whereas over-reporting is minimal for event households (2.1 percent for births and zero for marriages), it is very high for relatives and higher for parents than for siblings. Thus, over-reporting by parents reaches an extremely high 37.0 percent for marriages (mostly marriages occurring in 1972 or 1974) and 16.2 percent for births. The resulting total overcoverage biases, which are only slightly increased by the contribution of the multiplicity reporting variance, are thus high for both the multiplicity rules (7.7-11.6 percent) and offset the undercoverage biases to give total positive net biases. The overcoverage bias for the conventional rule is similar to the undercoverage bias for births (giving a net bias close to zero) and zero for marriages (giving a negative net bias).
The increase in bias resulting from the use of multiplicity counting rules (except for the full multiplicity rule for marriages) is offset by the decrease in variance (Table 1). The decrease from the variance of the conventional estimate is greater for births (about 50 percent for the full multiplicity rule and about 30 percent for the restricted rule) than for marriages (about 40 and 20 percent, respectively). The reduction of variance is due to the decrease in sampling variance, which is the major component in the variance. Thus, the relative sampling standard errors for the conventional rule, of 5.2 percent for births and of 6.0 percent for marriages, are reduced to 4.0 and 5.2 percent, respectively, for the restricted multiplicity rule, and to 3.4 and 4.2 percent, respectively, for the full multiplicity rule. The (simple) response error, however, increases with wider counting rules—from a relative response standard error of 0.9–1.0 percent for the conventional estimate to 1.7–2.1 percent for the restricted rule and to 2.2–2.6 percent for the full multiplicity rule.

Because of the different relative weights of the bias and variance components for different counting rules, the relative efficiencies of the multiplicity estimates (relative to the conventional estimate) decrease with increasing sample size. Calculations of the relative efficiencies, as functions of sample size, indicate that for marriages the full multiplicity rule estimates are always more efficient than the conventional rule estimates since both bias and variance are smaller than the conventional, and for births are more efficient for samples up to a size of about 19,000 (relative error of 2.5 percent). Because of its higher variance and bias, the restricted rule is less efficient than the full multiplicity rule for all sample sizes. Similarly, due to the high bias of the restricted multiplicity rule, it is also less efficient than the conventional rule, for marriages for all practical sample sizes and for births for all sample sizes greater than approximately 4,400 (relative error of 5.1 percent).

Thus, the limitations of the model and the estimates notwithstanding, there are clear indications that in this situation, for the ranges of sample sizes generally considered, the full multiplicity rule offers definite superiority over the conventional rule, for the same size sample. Cost considerations may reduce this superiority somewhat, although experience shows that the addition of reporting relatives and reports on multiplicity increases costs only marginally. The restricted multiplicity rule (conventional plus parents) is, however, counterindicated, primarily because of high overcoverage bias due to parents (especially for marriages). It might, therefore, be worthwhile to consider a different restricted counting rule based on conventional plus siblings (without parents). The structure of the present survey did not, however, allow this counting rule to be evaluated.

[Received November 1975. Revised May 1976.]

REFERENCES


