בס״ד

(1) **ניתוח הישרדות - תרגיל מס' 3 - פתרון**

1\. a) $\delta^* = 12 + 6 = 18$ cases סה״כ

$\tau^* = \{ .8 + 3.2 + 5.3 + 6.5 + 7.5 + 11.0$

Sample A → $+ 12.8 + 15.4 + 16.3 + 19.8 + 21.0 + 22.6 + 88(24.0) \}$

$+ \{ 1.3 + 2.5 + 5.7 + 6.8 + 8.8 + 11.5 + 94(12.0) \}$ ← תרומה של אלה שלא סבלו אירוע

Sample B

$= 2254.2 + 1164.6 = 3418.8$

> מה שמסומן τ^* כאן סימנו X^* בשיעורים

$\hat{\lambda} = 18/3418.8 = 0.005265$ cases/mo

$\hat{SE}(\hat{\lambda}) = \sqrt{\delta^*}/\tau^* = 0.001241$

$\hat{\Lambda}(24 \text{ mos}) = 24 \cdot (0.005265) = 0.12636$

$\hat{F}(24 \text{ mos}) = 1 - e^{-.12636} = 0.11870$

$\hat{S}(24 \text{ mos}) = e^{-.12636} = 0.8813$

b)

Month	δ^*	τ^*	$\hat{\lambda} = \delta^*/\tau^*$
1	1	199.8	.00501
2	1	198.3	.00504
3	1	197.5	.00506
4	1	196.2	.00510
5	0	196.0	0
6	2	195.0	.01026
7	2	193.3	.01035
8	1	191.5	.00522
9	1	190.8	.00524
10	0	190.0	0
11	1	190.0	.00524
12	1	188.5	.00531
13	1	93.8	.01066
14	0	93	0
15	0	93	0
16	1	92.4	.01082
17	1	91.3	.01095
18	0	91	0
19	0	91	0
20	1	90.8	.01101
21	1	90	.01111
22	1	88.6	.01129
23	0	88	0
24	0	88	0

כי האורך של כל קטע הינו חודש אחד

$\hat{\Lambda}(24 \text{ mos}) = \sum_{j=1}^{24} \hat{\lambda}_j (1) = 0.1279$

$\hat{F}(24 \text{ mos}) = 1 - e^{-.1279} = 0.1199$

$\hat{S}(24 \text{ mos}) = e^{-.1279} = 0.8799$

(2) c)

Event Time	N_k
.8	200
1.3	199
2.5	198
3.2	197
5.3	196
5.7	195
6.5	194
6.8	193
7.5	192
8.8	191
11.0	190
11.5	189
12.8	94
15.4	93
16.3	92
19.8	91
21.0	90
22.6	89

$$\tilde{\Lambda}(24 \text{ mos}) = \sum_k N_k^{-1}$$

$$= .12731$$

$$\hat{F}(24 \text{ mos}) = 1 - e^{-.12731}$$

$$= 0.1195$$

$$\hat{S}(24 \text{ mos}) = e^{-.12731} = 0.8805$$

(3)

$$\hat{S}_{\text{KM}}(24) = \prod_{l:\, T_{(l)} \le 24} \left(1 - \frac{d_l}{R(T_l)}\right)$$

סדר הכפלה ←

$$= \left(1-\tfrac{1}{200}\right)\left(1-\tfrac{1}{199}\right)\left(1-\tfrac{1}{198}\right)\left(1-\tfrac{1}{197}\right)\left(1-\tfrac{1}{196}\right)\left(1-\tfrac{1}{195}\right)\left(1-\tfrac{1}{194}\right)$$

$$\times \left(1-\tfrac{1}{193}\right)\left(1-\tfrac{1}{192}\right)\left(1-\tfrac{1}{191}\right)\left(1-\tfrac{1}{190}\right)\left(1-\tfrac{1}{189}\right)$$

$$\times \left(1-\tfrac{1}{94}\right)\left(1-\tfrac{1}{93}\right)\left(1-\tfrac{1}{92}\right)\left(1-\tfrac{1}{91}\right)\left(1-\tfrac{1}{90}\right)\left(1-\tfrac{1}{89}\right) = 0.8800$$

0.8 1.3 2.5 3.2 5.3 5.7 6.5

6.8 7.5 8.8 11.0 11.5

12.8 15.4 16.3 19.8 21.0 22.6

כי נשלם כאן הנבדק
במעקב כ' שלו.

2. $\delta^* = 11$ (מספר האירועים)

$\tau^* = 2+6+12+54+\cdots+168+173+181 = 2410$

$\hat{\lambda} = \delta^* / \tau^* = 0.004564$

$\hat{V}ar(\hat{\lambda}) = \delta^* / (\tau^*)^2 = 1.8939 \times 10^{-6}$

כ"ס ל- λ :

$$\hat{\lambda} \pm 1.96 \sqrt{\hat{V}ar(\hat{\lambda})}$$

$$0.004564 \pm (1.96)(0.001376)$$

$$[0.001867,\ 0.007262]$$

$$\hat{S}(144) = e^{-(144)(0.004564)}$$

$$= 0.5183$$

כ"ס:

$$[e^{-(144)(.007262)},\ e^{-(144)(.001867)}]$$

$$[0.3515,\ 0.7643]$$

דומה לרווחים שקיבלנו מקודם.

אפשר לבנות כ"ס חלופי ל- λ על סמך כ"ס ל- $\log \lambda$ מהצורה הבאה:

$$\log \hat{\lambda} \pm 1.96 \left(\frac{1}{\lambda}\right)\left(\hat{V}ar(\hat{\lambda})\right)^{1/2}$$ (ע"פ שיטת דלתה)

$$-5.3895 \pm 0.5910$$

$$[-5.9804,\ -4.7985]$$

זה מוביל לכ"ס הבא ל- λ :

$$[e^{-5.9804},\ e^{-4.7985}]$$

$$[0.002528,\ 0.008242]$$

מזה מגיעים לכ"ס הבא עבור $S(144)$:

$$[0.3052,\ 0.6949]$$

```
* Program to run the parametric models *;

options ls=80;

data indat;
infile 'd:\survival\mrfit.dat';
input age 1-2 dbp 3-5 sbp 6-8 race 9 diab 10 chol 11-13 cigs 14-15
  exyr 16-17 exmo 18-19 exday 20-21 fyrs 22-23 icd9 24-27
  dyr 28-29 dmo 30-31 dday 32-33 died 34;
exdate = mdy(exmo,exday,exyr);
ddate = mdy(dmo,dday,dyr);
fdate = mdy(12,31,90);
time = min(fdate,ddate) - exdate;
time = time / 365.25;

proc lifereg;
model time*died(0) = / dist=exponential;

proc lifereg;
model time*died(0) = / dist = weibull;

proc lifereg;
model time*died(0) = / dist = lognormal;

run;
```

שדרוג מומחים לשנים

The SAS System

The LIFEREG Procedure

Model Information

Data Set	WORK.INDAT
Dependent Variable	Log(time)
Censoring Variable	died
Censoring Value(s)	0
Number of Observations	5440
Noncensored Values	1989
Right Censored Values	3451
Left Censored Values	0
Interval Censored Values	0
Name of Distribution	EXPONENT
Log Likelihood	-5285.705847

Algorithm converged.

Analysis of Parameter Estimates

Variable	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq	Label
Intercept	1	3.59202	0.02242	25663.2812	<.0001	Intercept
Scale	0	1.00000	0			Extreme value scale

Lagrange Multiplier Statistics

Variable	Chi-Square	Pr > ChiSq
Scale	280.8776	<.0001

The SAS System

The LIFEREG Procedure

Model Information

Data Set	WORK.INDAT
Dependent Variable	Log(time)
Censoring Variable	died
Censoring Value(s)	0
Number of Observations	5440
Noncensored Values	1989
Right Censored Values	3451
Left Censored Values	0
Interval Censored Values	0
Name of Distribution	WEIBULL
Log Likelihood	-5202.499725

Algorithm converged.

Analysis of Parameter Estimates

Variable	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq	Label
Intercept	1	3.36131	0.02214	23043.9817	<.0001	Intercept
Scale	1	0.74919	0.01595			Extreme value scale

The SAS System

The LIFEREG Procedure

Model Information

Data Set	WORK.INDAT
Dependent Variable	Log(time)
Censoring Variable	died
Censoring Value(s)	0
Number of Observations	5440
Noncensored Values	1989
Right Censored Values	3451
Left Censored Values	0
Interval Censored Values	0
Name of Distribution	LNORMAL
Log Likelihood	-5296.048383

Algorithm converged.

Analysis of Parameter Estimates

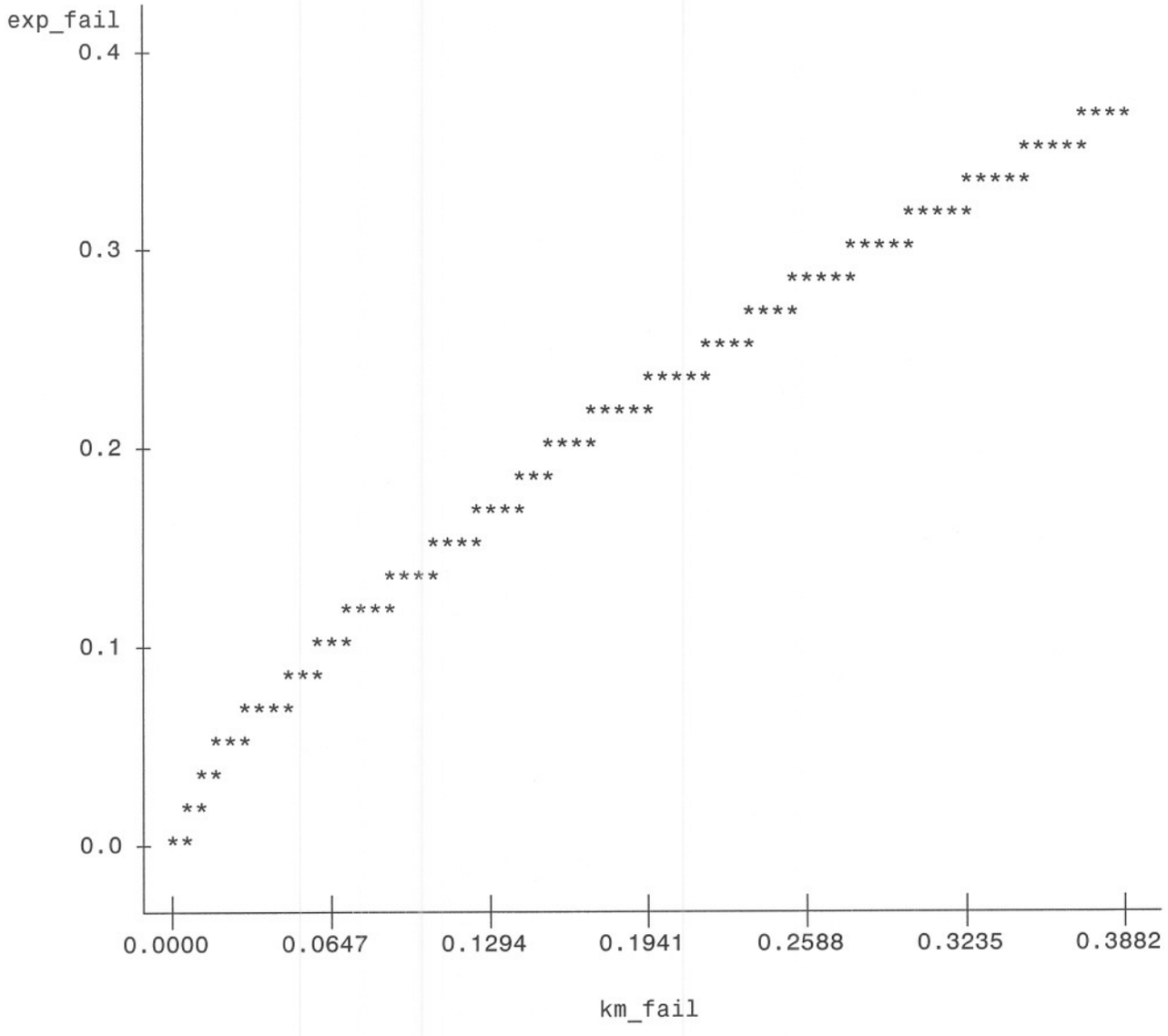
Variable	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq	Label
Intercept	1	3.27958	0.02746	14265.7131	<.0001	Intercept
Scale	1	1.35152	0.02417			Normal scale

```
* Program to take the results of the parametric
analysis and plot the parametric survival curves
vs. the KM survival curve. The relevant parameter
values are copied off of the printout from the
parametric model analyses. *;

options ls=80;

filename gsasfile 'd:\survival\scurv.ps';
goptions
  device=ps
  gsfname=gsasfile
  gsfmode=replace
  gsflen=80
  colors=(black);
symbol1 i=j line=1;
symbol2 i=j line=2;
axis1 length=5.5in label=('Kaplan-Meier');
axis2 length=5.5in label=('Parametric Model');

data indat;
infile 'd:\survival\mrfit.dat';
input age 1-2 dbp 3-5 sbp 6-8 race 9 diab 10 chol 11-13 cigs 14-15
  exyr 16-17 exmo 18-19 exday 20-21 fyrs 22-23 icd9 24-27
  dyr 28-29 dmo 30-31 dday 32-33 died 34;
exdate = mdy(exmo,exday,exyr);
ddate = mdy(dmo,dday,dyr);
fdate = mdy(12,31,90);
time = min(fdate,ddate) - exdate;
time = time / 365.25;

proc lifetest noprint notable outsurv=new method=km;
time time*died(0);

data new2; set new;
if _censor_=1 then delete;
if time=0 then delete;
exppar = 3.59202;
exp_lam = exp(-exppar);
weipar1 = 3.36131;
weipar2 = 0.74919;
wei_lam = exp(-weipar1);
wei_p = 1/weipar2;
exp_fail = 1 - exp(-exp_lam*time);
wei_fail = 1 - exp(-(wei_lam*time)**wei_p);
km_fail = 1 - survival;
lnmu = 3.27958;
lnsig = 1.35152;
ln_fail = probnorm((log(time)-lnmu)/lnsig);

keep time exp_fail wei_fail ln_fail km_fail;

proc plot;
```

```
plot exp_fail*km_fail='*';
plot wei_fail*km_fail='*';
plot ln_fail*km_fail='*';

data pdat;
set;
model = 'Exponential';
legend = 'Graph    ';
par_fail = exp_fail;
output;
legend = 'Reference';
par_fail = km_fail;
output;
model = 'Weibull';
legend = 'Graph    ';
par_fail = wei_fail;
output;
legend = 'Reference';
par_fail = km_fail;
output;
model = 'Lognormal';
legend = 'Graph    ';
par_fail = ln_fail;
output;
legend = 'Reference';
par_fail = km_fail;
output;

proc sort; by model time;

proc gplot; by model;
plot par_fail*km_fail=legend / haxis=axis1 vaxis=axis2 frame;

run;
```

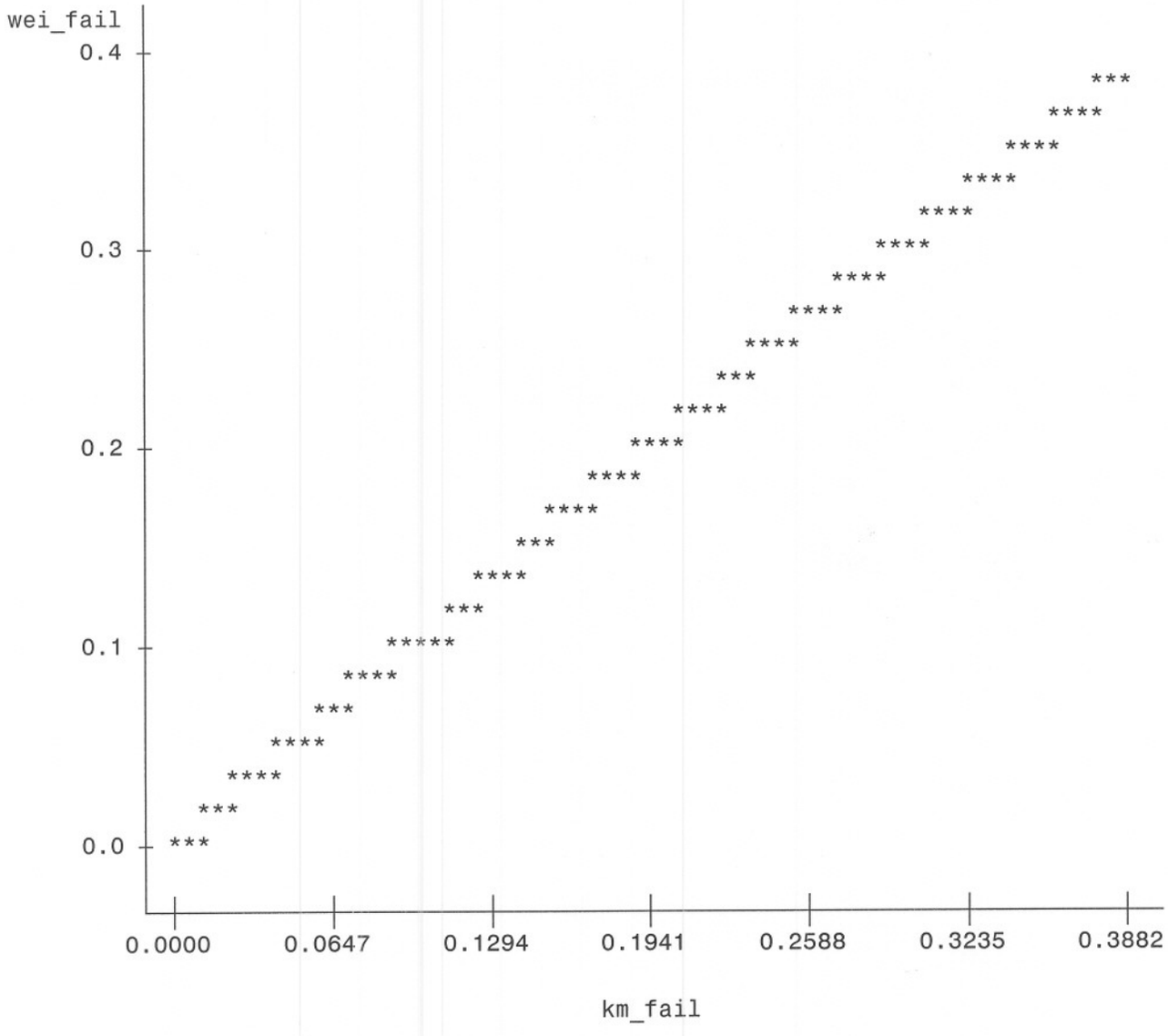
בגרפים רואים כי $\hat{S}_{LN}$, $\hat{S}_{Expo}$ טובים מ- $\hat{S}_{KM}$ בצורה ניכרת, בעוד ש- $\hat{S}_{Weibull}$ דומה מאוד ל- $\hat{S}_{KM}$.
לכן נראה כי Weibull הוא מודל טוב לנתונים אבל המודל המתאים והמודל הכלים ביותר אינם מודלים טובים לנתונים הללו.

Plot of exp_fail*km_fail. Symbol used is '*'.



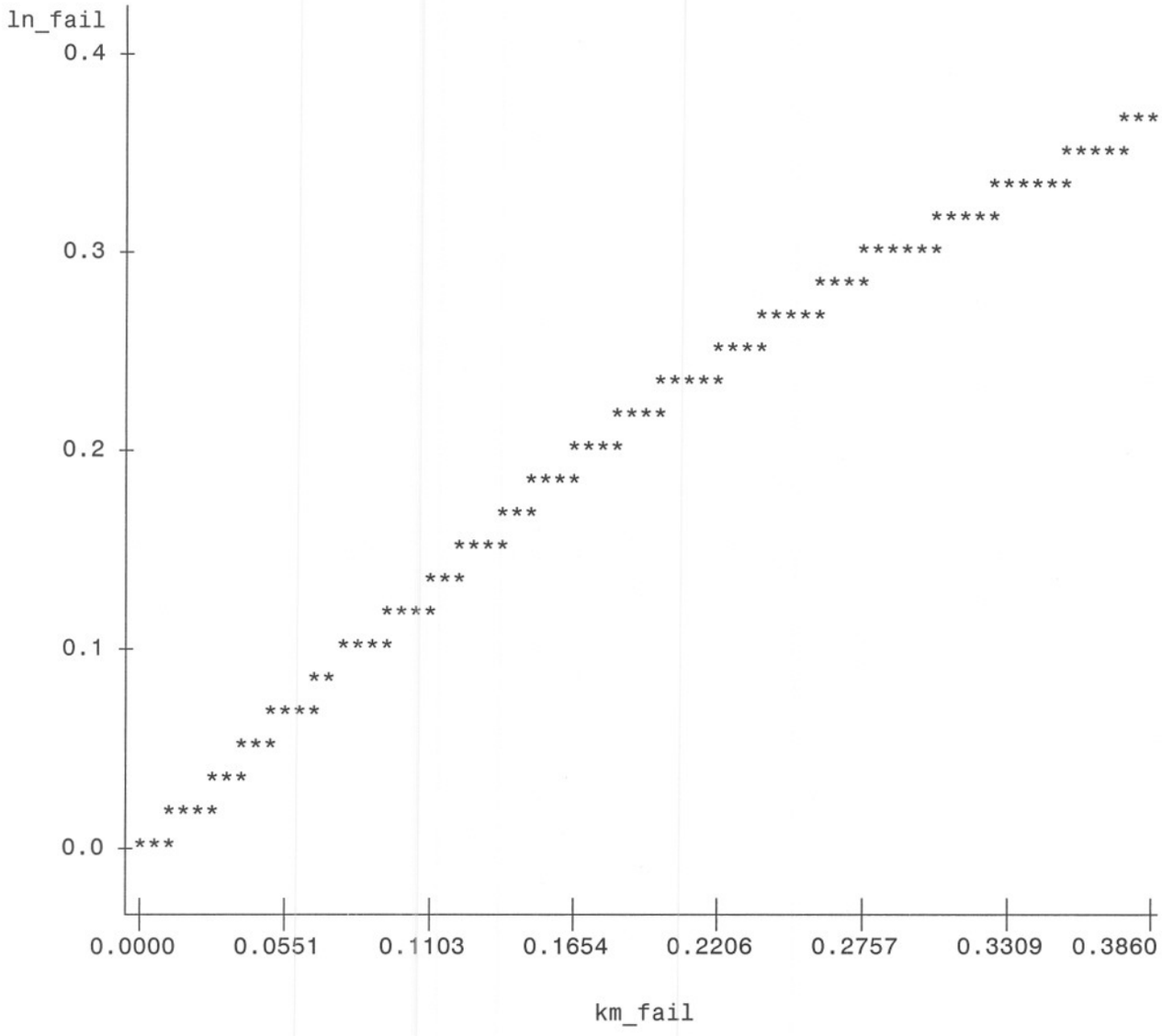
NOTE: 1606 obs hidden.

Plot of wei_fail*km_fail. Symbol used is '*'.



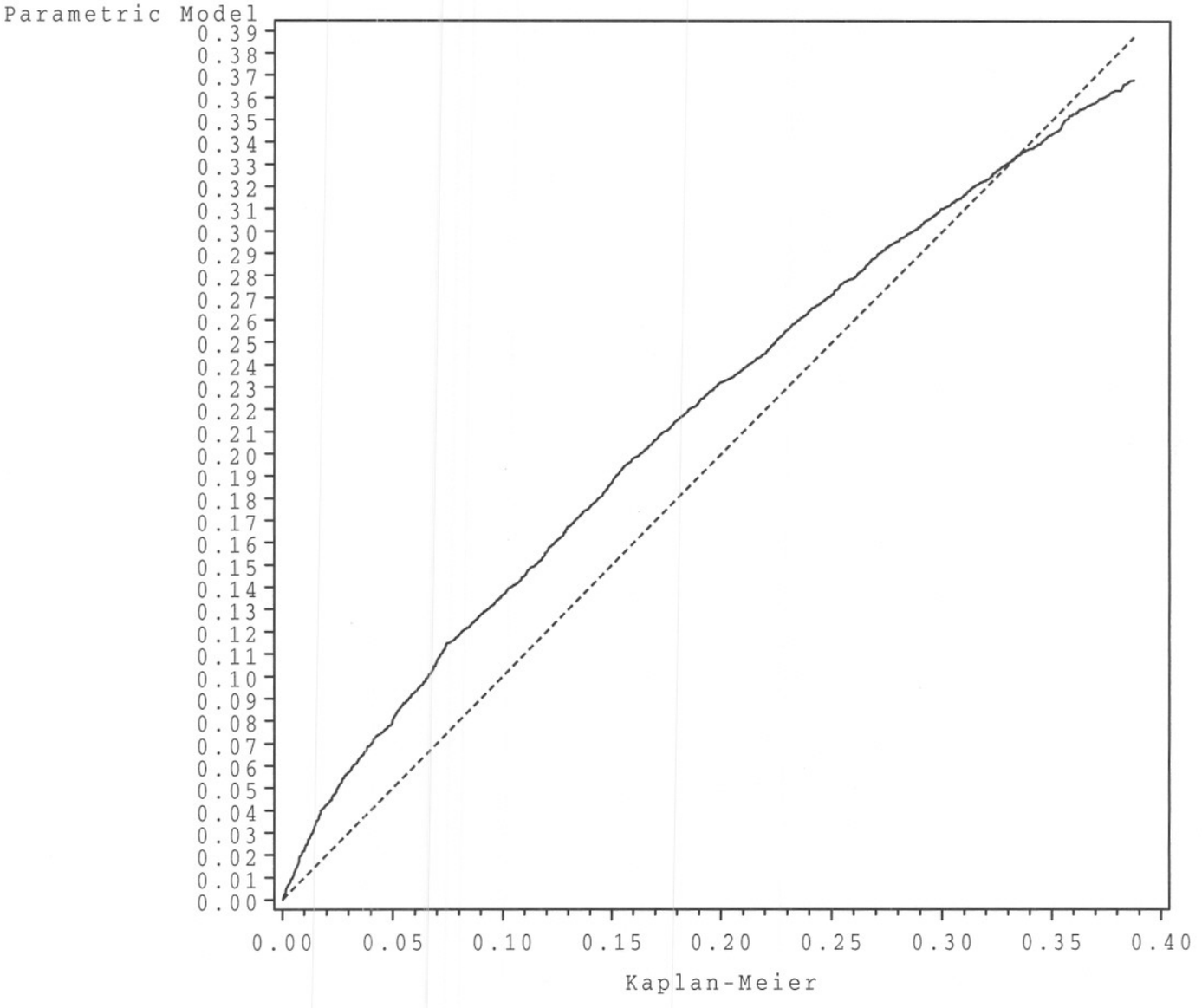
NOTE: 1605 obs hidden.

Plot of ln_fail*km_fail. Symbol used is '*'.

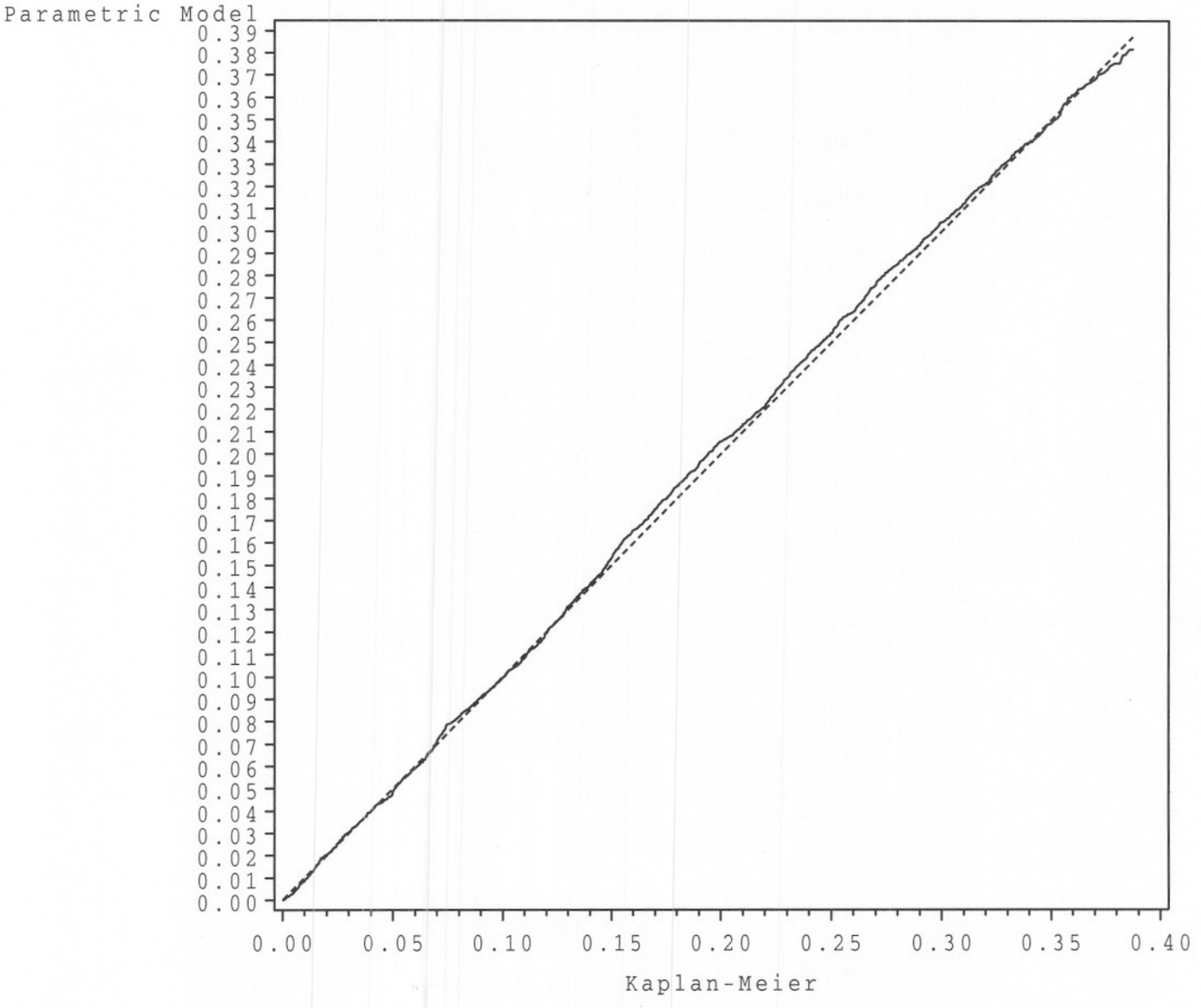


NOTE: 1603 obs hidden.

model=Exponential



model=Weibull



model=Lognormal