

יגוה נתונים ביטוחיג - גריון גרד'ס ל'ס

1. א. מנה ע' ז :

$$\begin{aligned} \frac{n_{01}}{n_{0.}} - \frac{n_{11}}{n_{1.}} &= \frac{1}{n_{0.}n_{1.}} (n_{01}n_{1.} - n_{11}n_{0.}) \\ &= \frac{1}{n_{0.}n_{1.}} [n_{01}(n_{10} + n_{11}) - n_{11}(n_{00} + n_{01})] \\ &= \frac{1}{n_{0.}n_{1.}} [n_{01}n_{10} + \cancel{n_{01}n_{11}} - n_{11}n_{00} - \cancel{n_{11}n_{01}}] \\ &= \frac{1}{n_{0.}n_{1.}} [n_{00}n_{11} - n_{01}n_{10}] \end{aligned}$$

מנה ע' ז (ג'ו) ג'ו ג'ו ע' :

$$\begin{aligned} \frac{n_{.0}}{n_{00}} \frac{n_{.1}}{n_{1.}} \left(\frac{1}{n_{0.}} + \frac{1}{n_{1.}} \right) &= n_{..} \\ &= \frac{n_{.0}n_{.1}}{n_{..}^2} \left(\frac{n_{0.} + n_{1.}}{n_{0.}n_{1.}} \right) = \frac{1}{n_{0.}} \frac{n_{.0}n_{.1}}{n_{0.}n_{1.}} \end{aligned}$$

$$\begin{aligned} z^2 &= \left(\frac{1}{n_{0.}n_{1.}} \right)^2 (n_{00}n_{11} - n_{01}n_{10})^2 / \left\{ \left(\frac{1}{n_{..}} \right) \left(\frac{n_{.0}n_{.1}}{n_{0.}n_{1.}} \right) \right\} \\ &= \frac{n_{..} (n_{00}n_{11} - n_{01}n_{10})^2}{(n_{0.}n_{1.}n_{.0}n_{.1})} = \chi^2 \end{aligned}$$

This proves that $(z')^2 = C$

ג. כ'סו ה'ג' ה'כ'ס (ע' כ'ס'סו) כ'ס'סו, כ'ס'סו, כ'ס'סו, כ'ס'סו
כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו כ'ס'סו

Here I used the notation N_{ij} for what is denoted in class by E_{ij} .

הוכחה (א) ו (ב) דומות

$$\begin{aligned}
 N_{11} + N_{12} &= n_{1.} & \text{and} & & n_{11} + n_{12} &= n_{1.} & \text{and} & & e' \\
 n_{12} - N_{12} &= -(n_{11} - N_{11}) & & & & & & & \text{and} \\
 & & & & & & & & \text{and} \\
 n_{21} - N_{21} &= -(n_{11} - N_{11}) & & & & & & & \\
 n_{22} - N_{22} &= -[-(n_{11} - N_{11})] = n_{11} - N_{11}
 \end{aligned}$$

$\chi^2 = \sum_{i,j} \frac{(n_{ij} - N_{ij})^2}{N_{ij}}$ 'sic

הוכחה (א) ו (ב) דומות

$$\chi^2 = \sum_{i,j=1}^2 \frac{(n_{ij} - N_{ij})^2}{N_{ij}}$$

$$= (n_{11} - N_{11})^2 \left[\frac{1}{N_{11}} + \frac{1}{N_{12}} + \frac{1}{N_{21}} + \frac{1}{N_{22}} \right]$$

$$= (n_{11} - N_{11})^2 \left[\frac{N_{11} + N_{12}}{N_{11} N_{12}} + \frac{N_{21} + N_{22}}{N_{21} N_{22}} \right]$$

$$= (n_{11} - N_{11})^2 \left[\frac{n_{1.}}{N_{11} N_{12}} + \frac{n_{2.}}{N_{21} N_{22}} \right]$$

$$= n_{1.}^2 (n_{11} - N_{11})^2 \left[\frac{1}{n_{1.} n_{1.} n_{1.} n_{2.}} + \frac{1}{n_{2.} n_{1.} n_{2.} n_{2.}} \right]$$

$$= (n_{11} - N_{11})^2 / \left[\frac{n_{1.} n_{2.} n_{1.} n_{2.}}{n_{1.}^3} \right]$$

This proves A=B

B = C - e n n o i ?

$$B = \frac{(n_{11} - E_{11})^2}{\frac{n_{00} \cdot n_{10} \cdot n_{01} \cdot n_{11}}{n_{..}^3}} = \frac{(n_{11} - \frac{n_{10} \cdot n_{01}}{n_{..}})^2}{\frac{n_{00} \cdot n_{10} \cdot n_{01} \cdot n_{11}}{n_{..}^3}}$$

$$= \frac{(n_{11} n_{..} - n_{10} n_{01})^2 / n_{..}^2}{n_{00} \cdot n_{10} \cdot n_{01} \cdot n_{11} / n_{..}^3}$$

$$= \frac{n_{..} (n_{11} n_{..} - n_{10} n_{01})^2}{n_{00} \cdot n_{10} \cdot n_{01} \cdot n_{11}}$$

→ 83

$$\begin{aligned} & n_{11} n_{..} - n_{10} n_{01} \\ &= n_{11} (n_{00} + n_{01} + n_{10} + n_{11}) - (n_{10} + n_{11})(n_{01} + n_{11}) \\ &= n_{11} n_{00} + \cancel{n_{11} n_{01}} + \cancel{n_{10} n_{11}} + \cancel{n_{11}^2} \\ &\quad - [n_{10} n_{01} + \cancel{n_{11} n_{01}} + \cancel{n_{10} n_{11}} + \cancel{n_{11}^2}] \\ &= n_{00} n_{11} - n_{01} n_{10} \end{aligned}$$

B = C 's n n o i ? s n i

ω odds ratio δ or δ

R
1.0 1.5 2.0 2.5 3.0

$\pi_{B/\bar{A}}$

0.01	1.00	1.51	2.02	2.54	3.06
0.05	1.00	1.54	2.11	2.71	3.35
0.10	1.00	1.59	2.25	3.00	3.86
0.15	1.00	1.65	2.43	3.40	4.64
0.20	1.00	1.71	2.67	4.00	6.00

כואים או ק'רר
 בין ω - R כאשר
 $\pi_{B/\bar{A}}$ נמוך ורפער
 ביניכם כאשר $\pi_{B/\bar{A}}$
 גבוה יותר

$$\log \hat{R} = \log \hat{\pi}_{B|A} - \log \hat{\pi}_{B|\bar{A}}$$

ג' (ד) זהו יחס p , $g'(p) = 1/p$ וזה $g(p) = \log p$ וזה

$$\begin{aligned} \text{Var}(\log \hat{\pi}_{B|A}) &= \left(\frac{1}{\pi_{B|A}}\right)^2 \text{Var}(\hat{\pi}_{B|A}) \\ &= \left(\frac{1}{\pi_{B|A}}\right)^2 \frac{\pi_{B|A}(1-\pi_{B|A})}{n_{1.}} \\ &= \frac{1-\pi_{B|A}}{\pi_{B|A} n_{1.}} = \frac{1}{n_{1.}} \frac{\pi_{\bar{B}|A}}{\pi_{B|A}} \end{aligned}$$

Here again, the notation differs from what I used in class, but it should be clear.

$$\hat{\text{Var}}(\log \hat{\pi}_{B|A}) = \frac{1}{n_{1.}} \frac{n_{10}/n_{1.}}{n_{11}/n_{1.}} = \frac{1}{n_{1.}} \frac{n_{10}}{n_{11}}$$

$$\text{Var}(\log \hat{\pi}_{B|\bar{A}}) = \frac{1}{n_{0.}} \frac{\pi_{\bar{B}|\bar{A}}}{\pi_{B|\bar{A}}} \quad \text{זהו יחס}$$

$$\hat{\text{Var}}(\log \hat{\pi}_{B|\bar{A}}) = \frac{1}{n_{0.}} \frac{n_{00}}{n_{01}}$$

$$\text{Var}(\log \hat{R}) = \frac{1}{n_{0.}} \frac{\pi_{\bar{B}|\bar{A}}}{\pi_{B|\bar{A}}} + \frac{1}{n_{1.}} \frac{\pi_{\bar{B}|A}}{\pi_{B|A}} \quad \text{זהו}$$

$$\hat{\text{Var}}(\log \hat{R}) = \frac{1}{n_{0.}} \frac{n_{00}}{n_{01}} + \frac{1}{n_{1.}} \frac{n_{10}}{n_{11}}$$

$$\hat{R} = \frac{180/200}{150/200} = 1.2$$

1 R - d o"n .5
(= 1/0.8333)

$$\log \hat{R} = \log 1.2 = 0.1823$$

אמריקן זאכען " (ה' ה' ס' ד')

$$\widehat{\text{Var}}(\log \hat{R}) = \frac{1}{200} \frac{50}{150} + \frac{1}{200} \frac{20}{180} = 0.002222$$

$$\widehat{\text{Var}}(\log \hat{R})^{1/2} = \sqrt{0.002222} = 0.04714$$

log R - d (95% סכום) o"n

$$0.1823 \pm (1.96)(0.04714) \Rightarrow [0.08993, 0.2747]$$

$$[e^{0.08993}, e^{0.2747}]$$

: R - d o"n

$$= [1.0941, 1.3162] = [(0.9140)^{-1}, (0.7598)^{-1}]$$

-SAS se אקאזאציע און אקאזאציע

ω ∫ 0"

$$\hat{\omega} = \frac{(50)(180)}{(150)(20)} = 3$$

$$\log \hat{\omega} = \log 3 = 1.0986$$

כפי הנראה בבירור

$$\widehat{\text{Var}}(\log \hat{\omega}) = \frac{1}{50} + \frac{1}{150} + \frac{1}{20} + \frac{1}{180} = 0.0822$$

$$[\widehat{\text{Var}}(\log \hat{\omega})]^{1/2} = \sqrt{0.0822} = 0.2867$$

∫ 0" log ω

$$1.0986 \pm (1.96)(0.2867) \Rightarrow [0.5366, 1.6606]$$

∫ 0" ω

$$[e^{0.5366}, e^{1.6606}] = [1.7102, 5.2526]$$

הוא 8.5%