

$$L = \prod_{jk} \left[\frac{e^{\delta_j + \eta_k}}{(1+e^\delta)(1+e^\eta)} \right]^{n_{0jk}} \times \prod_{jk} \left[\frac{e^{\delta_j + \eta_k + \beta_{1j} + \beta_{2k} + \beta_{3jk}}}{\sum e^{\dots}} \right]^{n_{1jk}}$$

$$l = \sum_{jk} n_{0jk} \left[\delta_j + \eta_k - \log(1+e^\delta) - \log(1+e^\eta) \right] + \sum_{jk} n_{1jk} \left[\delta_j + \eta_k + \beta_{1j} + \beta_{2k} + \beta_{3jk} - \log \sum_{j'k'} e^{\dots} \right]$$

$$\frac{\partial l}{\partial \delta} = n_{01.} - n_{0..} \frac{e^\delta}{1+e^\delta} + n_{11.} - n_{1..} \frac{\sum_{j'k'} j' e^{\dots}}{\sum_{j'k'} e^{\dots}}$$

$$\frac{\partial l}{\partial \eta} = n_{001} - n_{0..} \frac{e^\eta}{1+e^\eta} + n_{101} - n_{1..} \frac{\sum_{j'k'} k' e^{\dots}}{\sum_{j'k'} e^{\dots}}$$

$$\frac{\partial l}{\partial \beta_1} = n_{11.} - n_{1..} \frac{\sum_{j'k'} j' e^{\dots}}{\sum_{j'k'} e^{\dots}}$$

$$\frac{\partial l}{\partial \beta_2} = n_{101} - n_{1..} \frac{\sum_{j'k'} k' e^{\dots}}{\sum_{j'k'} e^{\dots}}$$

$$\frac{\partial l}{\partial \beta_3} = n_{111} - n_{1..} \frac{e^{\delta + \eta + \beta_1 + \beta_2 + \beta_3}}{\sum_{j'k'} e^{\dots}}$$